S1·8

The normal distribution

CHAPTER ASSIGNMENT · 213 MARKS

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. Give answers to 3 significant figures unless otherwise specified. Fractions are written a/b, with brackets where needed.

Practice questions from *Cambridge International AS & A Level Mathematics: Probability & Statistics 1 Coursebook* (Cambridge University Press, Dean Chalmers), Chapter 8 (The normal distribution), Exercises 8A to 8E, End-of-chapter review exercise 8, and Cross-topic review exercise 3, for classroom use. Where a question needs a diagram, the diagram is described in words or given as a table. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 8A · Continuous random variables

8A.1 The probability distributions for A and B are represented in the diagram.

[Diagram description: Two normal curves plotted on the same horizontal axis. Curve A has a higher peak and smaller spread, centered at μ_A to the left of Curve B. Curve B has a lower peak and larger spread, centered at μ_B to the right of Curve A.]

Indicate whether each of the following statements is true or false:

(a) $\mu_A > \mu_B$

(b) $\sigma_A < \sigma_B$

(c) A and B have the same range of values.

(d) $\sigma_A^2 = \sigma_B^2$

(e) At least half of the values in B are greater than μ_A.

(f) At most half of the values in A are less than μ_B.

8A.2 The diagram shows normal curves for the probability distributions of P and Q, that each contain n values.

[Diagram description: Two normal curves on the same axes. Curve P has a lower peak and larger spread, centered at μ_P to the left of Curve Q. Curve Q has a higher peak and smaller spread, centered at μ_Q to the right of Curve P.]

(a) Write down a statement comparing:

(i) σ_P and σ_Q

(ii) the median value for P and the median value for Q

(iii) the interquartile range for P and the interquartile range for Q.

(b) The datasets P and Q are merged to form a new dataset denoted by W.

(i) Describe the range of W.

(ii) Is the probability distribution for W a normal curve? Explain your answer.

(iii) Sketch onto a diagram a curved graph representing the probability distribution for W. Mark the relative positions of μ_P, μ_Q and μ_W along the horizontal axis.

8A.3 The distributions of the heights of 1000 women and of 1000 men both produce normal curves, as shown. The mean height of the women is 160 cm and the mean height of the men is 180 cm.

[Diagram description: Two identically shaped bell curves with equal peak heights and equal spread, one centered at 160 cm (women) and the other centered at 180 cm (men).]

The heights of these women and men are now combined to form a new set of data. Assuming that the combined heights also produce a normal curve, sketch the curve for the combined heights of the 2000 women and men.

8A.4 Probability distributions for the quantity of apple juice in 500 apple juice tins and for the quantity of peach juice in 500 peach juice tins are both represented by normal curves. The mean quantity of apple juice is 340 ml with variance 4 ml^2, and the mean quantity of peach juice is 340 ml with standard deviation 4 ml.

[Diagram description: A normal curve centered at 340 ml representing the apple juice distribution.]

(a) Sketch onto a diagram the normal curve for the quantity of peach juice in the peach juice tins.

(b) Describe the curves' differences and similarities.

8A.5 The masses of 444 newborn babies in the USA and 888 newborn babies in the UK both produce normal curves. For the USA babies, $\mu = 3.4$ kg and $\sigma = 200$ g; for the UK babies, $\mu = 3.3$ kg and $\sigma^2 = 36\,100$ g^2.

(a) On a single diagram, sketch and label these two normal curves.

(b) Describe the curves' differences and similarities.

8A.6 The values in two datasets, whose probability distributions are both normal curves, are summarised by the following totals:

$\Sigma x^2 = 35\,000$, $\Sigma x = 12\,000$ and $n = 5000$.

$\Sigma y^2 = 72\,000$, $\Sigma y = 26\,000$ and $n = 10\,000$.

(a) Show that the centre of the curve for y is located to the right of the curve for x.

(b) On the same diagram, sketch a normal curve for each dataset.

Exercise 8B · The standard normal distribution

8B.1 Given that $Z \sim N(0, 1)$, find the following probabilities correct to 3 significant figures:

(a) $P(Z < 0.567)$

(b) $P(Z \leq 2.468)$

(c) $P(Z > -1.53)$

(d) $P(Z \geq -0.077)$

(e) $P(Z > 0.817)$

(f) $P(Z \geq 2.009)$

(g) $P(Z < -1.75)$

(h) $P(Z \leq -0.013)$

(i) $P(Z < 1.96)$

(j) $P(Z > 2.576)$

8B.2 The random variable Z is normally distributed with mean 0 and variance 1. Find the following probabilities, correct to 3 significant figures:

(a) $P(1.5 < Z < 2.5)$

(b) $P(0.046 < Z < 1.272)$

(c) $P(1.645 < Z < 2.326)$

(d) $P(-2.807 < Z < -1.282)$

(e) $P(-1.777 < Z < -0.746)$

(f) $P(-1.008 < Z < -0.337)$

(g) $P(-1.2 < Z < 1.2)$

(h) $P(-1.667 < Z < 2.667)$

(i) $P(-3/4 \leq Z < 8/5)$

(j) $P(\sqrt{2} \leq Z < \sqrt{5})$

8B.3 Given that $Z \sim N(0, 1)$, find the value of k, given that:

(a) $P(Z < k) = 0.9087$

(b) $P(Z < k) = 0.5442$

(c) $P(Z > k) = 0.2743$

(d) $P(Z > k) = 0.0298$

(e) $P(Z < k) = 0.25$

(f) $P(Z < k) = 0.3552$

(g) $P(Z > k) = 0.9296$

(h) $P(Z > k) = 0.648$

(i) $P(-k < Z < k) = 0.9128$

(j) $P(-k < Z < k) = 0.6994$

8B.4 Find the value of c in each of the following where Z has a normal distribution with $\mu = 0$ and $\sigma^2 = 1$:

(a) $P(c < Z < 1.638) = 0.2673$

(b) $P(c < Z < 2.878) = 0.4968$

(c) $P(1 < Z < c) = 0.1408$

(d) $P(0.109 < Z < c) = 0.35$

(e) $P(c < Z < 2) = 0.6687$

(f) $P(c < Z < 1.85) = 0.9516$

(g) $P(-1.221 < Z < c) = 0.888$

(h) $P(-0.674 < Z < c) = 0.725$

(i) $P(-2.63 < Z < c) = 0.6861$

(j) $P(-2.7 < Z < c) = 0.0252$

Exercise 8C · Standardising normal distributions

8C.1 Standardise the appropriate value(s) of the normal variable X represented in each diagram, and find the required probabilities correct to 3 significant figures:

(a) Find $P(X \leq 11)$, given that $X \sim N(8, 25)$. [Diagram shows normal curve centered at 8 with shaded region to the left of 11.]

(b) Find $P(X < 69.1)$, given that $X \sim N(72, 11)$. [Diagram shows normal curve centered at 72 with shaded region to the left of 69.1.]

(c) Find $P(3 < X < 7)$, given that $X \sim N(5, 5)$. [Diagram shows normal curve centered at 5 with shaded region between 3 and 7.]

8C.2 Calculate the required probabilities correct to 3 significant figures:

(a) Find $P(X \leq 9.7)$ and $P(X > 9.7)$, given that $X \sim N(6.2, 6.25)$.

(b) Find $P(X \leq 5)$ and $P(X > 5)$, given that $X \sim N(3, 49)$.

(c) Find $P(X > 33.4)$ and $P(X \leq 33.4)$, given that $X \sim N(37, 4)$.

(d) Find $P(X < 13.5)$ and $P(X \geq 13.5)$, given that $X \sim N(20, 15)$.

(e) Find $P(X > 91)$ and $P(X \leq 91)$, given that $X \sim N(80, 375)$.

(f) Find $P(1 \leq X < 21)$, given that $X \sim N(11, 25)$.

(g) Find $P(2 \leq X < 5)$, given that $X \sim N(3, 7)$.

(h) Find $P(6.2 \geq X \geq 8.8)$, given that $X \sim N(7, 1.44)$. [Read carefully.]

(i) Find $P(26 \leq X < 28)$, given that $X \sim N(25, 6)$.

(j) Find $P(8 \leq X < 10)$, given that $X \sim N(12, 2.56)$.

8C.3 (a) Find a, given that $X \sim N(30, 16)$ and that $P(X \leq a) = 0.8944$.

(b) Find b, given that $X \sim N(12, 4)$ and that $P(X \leq b) = 0.9599$.

(c) Find c, given that $X \sim N(23, 9)$ and that $P(X > c) = 0.9332$.

(d) Find d, given that $X \sim N(17, 25)$ and that $P(X > d) = 0.0951$.

(e) Find e, given that $X \sim N(100, 64)$ and that $P(X \geq e) = 0.95$.

8C.4 (a) Find f, given that $X \sim N(10, 7)$ and that $P(f \leq X < 13.3) = 0.1922$.

(b) Find g, given that $X \sim N(45, 50)$ and that $P(g \leq X < 55) = 0.5486$.

(c) Find h, given that $X \sim N(7, 2)$ and that $P(8 \leq X < h) = 0.216$.

(d) Find j, given that $X \sim N(20, 11)$ and that $P(j \leq X < 22) = 0.5$.

8C.5 X is normally distributed with mean 4 and variance 6. Find the probability that X takes a negative value.

8C.6 Given that $X \sim N(\mu, (4/9)\mu^2)$ where $\mu > 0$, find $P(X < 2\mu)$.

8C.7 If $T \sim N(10, \sigma^2)$ and $P(T > 14.7) = 0.04$, find the value of σ.

8C.8 It is given that $V \sim N(\mu, 13)$ and $P(V < 15) = 0.75$. Find the value of μ.

8C.9 The variable $W \sim N(\mu, \sigma^2)$. Given that $\mu = 4\sigma$ and $P(W < 83) = 0.95$, find the value of μ and of σ.

8C.10 X has a normal distribution in which $\sigma = \mu - 30$ and $P(X \geq 12) = 0.9$. Find the value of μ and of σ.

8C.11 The variable $Q \sim N(\mu, \sigma^2)$. Given that $P(Q < 1.288) = 0.281$ and $P(Q < 6.472) = 0.591$, find the value of μ and of σ, and calculate $P(4 \leq Q < 5)$.

8C.12 For the variable $V \sim N(\mu, \sigma^2)$, it is given that $P(V < 8.4) = 0.7509$ and $P(V > 9.2) = 0.1385$. Find the value of μ and of σ, and calculate $P(V \leq 10)$.

8C.13 Find the value of μ and of σ and calculate $P(W > 6.48)$ for the variable $W \sim N(\mu, \sigma^2)$, given that $P(W \geq 4.75) = 0.6858$ and $P(W \leq 2.25) = 0.0489$.

8C.14 X has a normal distribution, such that $P(X > 147.0) = 0.0136$ and $P(X \leq 59.0) = 0.0038$. Use this information to calculate the probability that $80.0 \leq X < 130.0$.

Exercise 8D · Modelling with the normal distribution

8D.1 The length of a bolt produced by a machine is normally distributed with mean 18.5 cm and variance 0.7 cm^2. Find the probability that a randomly selected bolt is less than 18.85 cm long.

8D.2 The waiting times, in minutes, for patients at a clinic are normally distributed with mean 13 and variance 16.

(a) Calculate the probability that a randomly selected patient has to wait for more than 16.5 minutes.

(b) Last month 468 patients attended the clinic. Calculate an estimate of the number who waited for less than 9 minutes.

8D.3 Tomatoes from a certain producer have masses which are normally distributed with mean 90 grams and standard deviation 17.7 grams. The tomatoes are sorted into three categories by mass, as follows: Small: under 80 g; Medium: 80 g to 104 g; Large: over 104 g.

(a) Find, correct to 2 decimal places, the percentage of tomatoes in each of the three categories.

(b) Find the value of k such that $P(k \leq X < 104) = 0.75$, where X is the mass of a tomato in grams.

8D.4 The heights, in metres, of the trees in a forest are normally distributed with mean μ and standard deviation 3.6. Given that 75% of the trees are less than 10 m high, find the value of μ.

8D.5 The mass of a certain species of fish caught at sea is normally distributed with mean 5.73 kg and variance 2.56 kg^2. Find the probability that a randomly selected fish caught at sea has a mass that is:

(a) less than 6.0 kg

(b) more than 3.9 kg

(c) between 7.0 and 8.0 kg.

8D.6 The distance that children at a large school can hop in 15 minutes is normally distributed with mean 199 m and variance 3700 m^2.

(a) Calculate an estimate of b, given that only 25% of the children hopped further than b metres.

(b) Find an estimate of the interquartile range of the distances hopped.

8D.7 The daily percentage change in the value of a company's shares is expected to be normally distributed with mean 0 and standard deviation 0.51. On how many of the next 365 working days should the company expect the value of its shares to fall by more than 1%?

8D.8 The masses, w grams, of a large sample of apples are normally distributed with mean 200 and variance 169. Given that the masses of 3413 apples are in the range $187 < w < 213$, calculate an estimate of the number of apples in the sample.

8D.9 The ages of the children in a gymnastics club are normally distributed with mean 15.2 years and standard deviation σ. Find the value of σ given that 30.5% of the children are less than 13.5 years of age.

8D.10 The speeds, in km h^{-1}, of vehicles passing a particular point on a rural road are normally distributed with mean μ and standard deviation 20. Find the value of μ and find what percentage of the vehicles are being driven at under 80 km h^{-1}, given that 33% of the vehicles are being driven at over 100 km h^{-1}.

8D.11 Coffee beans are packed into bags by the workers on a farm, and each bag claims to contain 200 g. The actual mass of coffee beans in a bag is normally distributed with mean 210 g and standard deviation σ. The farm owner informs the workers that they must repack any bag containing less than 200 g of coffee beans. Find the value of σ, given that 0.5% of the bags must be repacked.

8D.12 Colleen exercises at home every day. The length of time she does this is normally distributed with mean 12.8 minutes and standard deviation σ. She exercises for more than 15 minutes on 42 days in a year of 365 days.

(a) Calculate the value of σ.

(b) On how many days in a year would you expect Colleen to exercise for less than 10 minutes?

8D.13 The times taken by 15-year-olds to solve a certain puzzle are normally distributed with mean μ and standard deviation 7.42 minutes.

(a) Find the value of μ, given that three-quarters of all 15-year-olds take over 20 minutes to solve the puzzle.

(b) Calculate an estimate of the value of n, given that 250 children in a random sample of n 15-year-olds fail to solve the puzzle in less than 30 minutes.

8D.14 The lengths, X cm, of the leaves of a particular species of tree are normally distributed with mean μ and variance σ^2.

(a) Find $P(\mu - \sigma < X < \mu + \sigma)$.

(b) Find the probability that a randomly selected leaf from this species has a length that is more than 2 standard deviations from the mean.

(c) Find the value of μ and of σ, given that $P(X < 7.5) = 0.75$ and $P(X < 8.5) = 0.90$.

8D.15 The time taken in seconds for Ginger's computer to open a specific large document is normally distributed with mean 9 and variance 5.91.

(a) Find the probability that it takes exactly 5 seconds or more to open the document.

(b) Ginger opens the document on her computer on n occasions. The probability that it fails to open in less than exactly 5 seconds on at least one occasion is greater than 0.5. Find the least possible value of n.

8D.16 The masses of all the different pies sold at a market are normally distributed with mean 400 g and standard deviation 61 g. Find the probability that:

(a) the mass of a randomly selected pie is less than 425 g

(b) 4 randomly selected pies all have masses of less than 425 g

(c) exactly 7 out of 10 randomly selected pies have masses of less than 425 g.

8D.17 The height of a female university student is normally distributed with mean 1.74 m and standard deviation 12.3 cm. Find the probability that:

(a) a randomly selected female student is between 1.71 and 1.80 metres tall

(b) 3 randomly selected female students are all between 1.71 and 1.80 m tall

(c) exactly 15 out of 50 randomly selected female students are between 1.71 and 1.80 metres tall.

Exercise 8E · The normal approximation to the binomial distribution

8E.1 Decide whether or not each of the following binomial distributions can be well-approximated by a normal distribution. For those that can, state the values of the parameters μ and σ^2. For those that cannot, state the reason:

(a) B(20, 0.6)

(b) B(30, 0.95)

(c) B(40, 0.13)

(d) B(50, 0.06)

8E.2 Find the smallest possible value of n for which the following binomial distributions can be well-approximated by a normal distribution:

(a) B(n, 0.024)

(b) B(n, 0.15)

(c) B(n, 0.52)

(d) B(n, 0.7)

8E.3 Describe the binomial distribution that can be approximated by the normal distribution N(14, 10.5).

8E.4 By first evaluating np and npq, use a suitable approximation and continuity correction to find $P(X < 75)$ for the discrete random variable $X \sim B(100, 0.7)$.

8E.5 The discrete random variable $Y \sim B(50, 0.6)$. Use a suitable approximation and continuity correction to find $P(Y > 26)$.

8E.6 A biased coin is tossed 160 times. The number of heads obtained, H, follows a binomial distribution where $E(H) = 100$. Find:

(a) the value of p and the variance of H

(b) the approximate probability of obtaining more than 110 heads.

8E.7 One card is selected at random from each of 40 packs. Each pack contains 52 cards and includes 13 clubs. Let C be the number of clubs selected from the 40 packs.

(a) Show that the variance of C is 7.5.

(b) Obtain an approximation for the value of P(C < 8), and justify the use of this approximation.

8E.8 In a large survey, 55% of the people questioned are in full-time employment. In a random sample of 80 of these people, find:

(a) the expected number in full-time employment

(b) the standard deviation of the number in full-time employment

(c) the approximate probability that fewer than half of the sample are in full-time employment.

8E.9 A company manufactures rubber and plastic washers in the ratio 4 : 1. The washers are randomly packed into boxes of 25.

(a) Find the probability that a randomly selected box contains:

(i) exactly 21 rubber washers

(ii) exactly 10 plastic washers.

(b) A retail pack contains 2000 washers. Find the expectation and variance of the number of rubber washers in a retail pack.

(c) Using a suitable approximation, find the probability that a retail pack contains at most 1620 rubber washers.

8E.10 In a certain town, 63% of homes have an internet connection.

(a) In a random sample of 20 homes in this town, find the probability that:

(i) exactly 15 have an internet connection

(ii) exactly nine do not have an internet connection.

(b) Use a suitable approximation to find the probability that more than 65% of a random sample of 600 homes in this town have an internet connection.

8E.11 17% of the people interviewed in a survey said they watch more than two hours of TV per day. A random sample of 300 of those who were interviewed is taken. Find an approximate value for the probability that at least one-fifth of those in the sample watch more than two hours of TV per day.

8E.12 An opinion poll was taken before an election. The table shows the percentage of voters who said they would vote for parties A, B and C:

Party	A	B	C
Votes (%)	36	41	23

Find an approximation for the probability that, in a random sample of 120 of these voters:

(a) exactly 50 said they would vote for party B

(b) more than 70 but fewer than 90 said they would vote for party B or party C.

8E.13 Boxes containing 24 floor tiles are loaded into vans for distribution. In a load of 80 boxes there are, on average, three damaged floor tiles. Find, approximately, the probability that:

(a) there are more than 65 damaged tiles in a load of 1600 boxes

(b) in five loads, each containing 1600 boxes, exactly three loads contain more than 65 damaged tiles.

8E.14 It is known that 2% of the cheapest memory sticks on the market are defective.

(a) In a random sample of 400 of these memory sticks, find approximately the probability that at least five but at most 11 are defective.

(b) Ten samples of 400 memory sticks are tested. Find an approximate value for the probability that there are fewer than 12 defective memory sticks in more than seven of the samples.

8E.15 Randomly selected members of the public were asked whether they approved of plans to build a new sports centre and 57% said they approved. Find approximately the probability that more than 75 out of 120 people said they approved, given that at least 60 said they approved.

8E.16 A fair coin is tossed 400 times. Given that it shows a head on more than 205 occasions, find an approximate value for the probability that it shows a head on fewer than 215 occasions.

8E.17 An ordinary fair die is rolled 450 times. Given that a 6 is rolled on fewer than 80 occasions, find approximately the probability that a 6 is rolled on at least 70 occasions.

End-of-chapter review exercise 8

R8.1 A continuous random variable, X, has a normal distribution with mean 8 and standard deviation σ. Given that $P(X > 5) = 0.9772$, find $P(X < 9.5)$. [3]

R8.2 The variable Y is normally distributed. Given that $10\sigma = 3\mu$ and $P(Y < 10) = 0.75$, find $P(Y \geq 6)$. [4]

R8.3 In Scotland, in November, on average 80% of days are cloudy. Assume that the weather on any one day is independent of the weather on other days.

(i) Use a normal approximation to find the probability of there being fewer than 25 cloudy days in Scotland in November (30 days). [4]

(ii) Give a reason why the use of a normal approximation is justified. [1]

(Cambridge International AS & A Level Mathematics 9709 Paper 62 Q2 June 2011)

R8.4 At a store, it is known that 1 out of every 9 customers uses a gift voucher in part-payment for purchases. A randomly selected sample of 72 customers is taken. Use a suitable approximation and continuity correction to find the probability that at most 6 of these customers use a gift voucher in part-payment for their purchases. [5]

R8.5 A survey shows that 54% of parents believe mathematics to be the most important subject that their children study. Use a suitable approximation to find the probability that at least 30 out of a sample of 50 parents believe mathematics to be the most important subject studied. [5]

R8.6 Two normally distributed continuous random variables are X and Y. It is given that X ~ N(1.5, 0.2^2) and that Y ~ N(2.0, 0.5^2). On the same diagram, sketch graphs showing the probability density functions of X and of Y. Indicate the line of symmetry of each clearly labelled graph. [3]

R8.7 The random variable X is such that X ~ N(82, 126).

(i) A value of X is chosen at random and rounded to the nearest whole number. Find the probability that this whole number is 84. [3]

(ii) Five independent observations of X are taken. Find the probability that at most one of them is greater than 87. [4]

(iii) Find the value of k such that $P(87 < X < k) = 0.3$. [5]

(Cambridge International AS & A Level Mathematics 9709 Paper 63 Q5 November 2012)

R8.8 (a) A petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.

(i) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres. [4]

The daily sales at another petrol station are X litres, where X is normally distributed with mean m and standard deviation 560. It is given that $P(X > 8000) = 0.122$.

(ii) Find the value of m. [3]

(iii) Find the probability that daily sales at this petrol station exceed 8000 litres on fewer than 2 of 6 randomly chosen days. [3]

(b) The random variable Y is normally distributed with mean μ and standard deviation σ. Given that σ = (2/3)μ, find the probability that a random value of Y is less than 2μ. [3]

(Cambridge International AS & A Level Mathematics 9709 Paper 62 Q7 November 2015)

R8.9 V and W are continuous random variables. V ~ N(9, 16) and W ~ N(6, σ^2). Find the value of σ, given that P(W < 8) = 2 × P(V < 8). [4]

R8.10 The masses, in kilograms, of 'giant Botswana cabbages' have a normal distribution with mean μ and standard deviation 0.75. It is given that 35.2% of the cabbages have a mass of less than 3 kg. Find the value of μ and the percentage of cabbages with masses of less than 3.5 kg. [5]

R8.11 The ages of the vehicles owned by a large fleet-hire company are normally distributed with mean 43 months and standard deviation σ. The probability that a randomly chosen vehicle is more than 4 1/6 years old is 0.28. Find what percentage of the company's vehicles are less than two years old. [5]

R8.12 The weights, X grams, of bars of soap are normally distributed with mean 125 grams and standard deviation 4.2 grams.

(i) Find the probability that a randomly chosen bar of soap weighs more than 128 grams. [3]

(ii) Find the value of k such that $P(k < X < 128) = 0.7465$. [4]

(iii) Five bars of soap are chosen at random. Find the probability that more than two of the bars each weigh more than 128 grams. [4]

(Cambridge International AS & A Level Mathematics 9709 Paper 62 Q7 November 2009)

R8.13 Crates of tea should contain 200 kg, but it is known that 1 out of 45 crates, on average, is underweight. A sample of 630 crates is selected at random.

(a) Find the probability that more than 12 but fewer than 17 crates are underweight. [5]

(b) Given that more than 12 but fewer than 17 crates are underweight, find the probability that more than 14 crates are underweight. [5]

R8.14 Once a week, Haziq rows his boat from the island where he lives to the mainland. The journey time, X minutes, is normally distributed with mean μ and variance σ^2.

(a) Given that $P(20 \leq X < 30) = 0.32$ and that $P(X < 20) = 0.63$, find the values of μ and σ^2. [4]

(b) The time taken for Haziq to row back home, Y minutes, is normally distributed and $P(Y < 20) = 0.6532$. Given that the variances of X and Y are equal, calculate:

(i) the mean time taken by Haziq to row back home [3]

(ii) the expected number of days over a period of five years (each of 52 weeks) on which Haziq takes more than 25 minutes to row back home. [3]

R8.15 The time taken, T seconds, to open a graphics programme on a computer is normally distributed with mean 20 and standard deviation σ. Given that $P(T > 13 \mid T \leq 27) = 0.8$, find the value of:

(a) σ [5]

(b) k for which $P(T > k) = 0.75$. [3]

R8.16 A law firm has found that their assistants make, on average, one error on every 36 pages that they type. A random sample of 90 typed documents, with a mean of 62 pages per document, is selected. Given that there are more than 140 typing errors in these documents, find an estimate of the probability that there are fewer than 175 typing errors. [6]

Cross-topic review exercise 3

X3.1 (a) The following table shows the probability distribution for the random variable X:

x	0	1	2	3
P(X = x)	1/k	3/10	3/20	1/20

(i) Show that k = 2. [2]

(ii) Calculate E(X) and Var(X). [3]

(iii) Find the probability that two independent observations of X have a sum of less than 6. [2]

(b) The following table shows the probability distribution for the random variable Y:

y	0	1	2	3
P(Y = y)	0.1	0.2	0.3	0.4

If one independent observation of each random variable is made, find the probability that X + Y = 3. [3]

X3.2 The random variable X has a geometric distribution such that P(X = 2) / P(X = 5) = 3 3/8. Find P(X ≤ 3). [3]

X3.3 The variable X has a normal distribution with mean μ and standard deviation σ. Given that P(X < 32.83) = 0.834 and that P(X ≥ 27.45) = 0.409, find the value of μ and of σ. [4]

X3.4 The length of time, in seconds, that it takes to transfer a photograph from a camera to a computer can be modelled by a normal distribution with mean 4.7 and variance 0.7225. Find the probability that a photograph can be transferred in less than 3 seconds. [3]

X3.5 The mass of a berry from a particular type of bush is normally distributed with mean 7.08 grams and standard deviation σ. It is known that 5% of the berries have a mass of exactly 12 grams or more.

(a) Find the value of σ. [3]

(b) Find the proportion of berries that have a mass of between 6 and 8 grams. [3]

X3.6 The time taken, in minutes, to fit a new windscreen to a car is normally distributed with mean μ and standard deviation 16.32. Given that three-quarters of all windscreens are fitted in less than 45 minutes, find:

(a) the value of μ [3]

(b) the proportion of windscreens that are fitted in 35 to 40 minutes. [3]

X3.7 The mid-day wind speed, in knots, at a coastal resort is normally distributed with mean 12.8 and standard deviation σ.

(a) Given that 15% of the recorded wind speeds are less than 10 knots, find the value of σ. [3]

(b) Calculate the probability that exactly two out of 10 randomly selected recordings are less than 10 knots. [3]

(c) Using a suitable approximation, calculate an estimate of the probability that at least 13 out of 100 randomly selected recordings are over 15.5 knots. [4]

X3.8 A technical manual contains 10 pages of text, 7 pages of diagrams and 3 pages of colour illustrations. Four different pages are selected at random from the manual. Let X be the number of pages of colour illustrations selected.

(a) Draw up the probability distribution table for X. [4]

(b) Find:

(i) E(X) [2]

(ii) the probability that fewer than two pages with colour illustrations are selected, given that at least one page with colour illustrations is selected. [2]

X3.9 A fair six-sided die is numbered 1, 1, 2, 3, 5 and 8. The die is rolled twice and the two numbers obtained are added together to give the score, X.

(a) Find E(X). [4]

(b) Given that the first number rolled is odd, find the probability that X is an even number. [2]

X3.10 The following table shows the probability distribution table for the random variable Q:

q	1	2	3
P(Q = q)	(x − 2)/(x + 1)	5/18	(x − 3)/(x + 4)

(a) Find the value of x. [3]

(b) Evaluate Var(Q). [2]

X3.11 Research shows that 17% of children are absent from school on at least five days during winter because of ill health. A random sample of 55 children is taken.

(a) Find the probability that exactly 10 of the children in the sample are absent from school on at least five days during winter because of ill health. [2]

(b) Use a suitable approximation to find the probability that at most seven children in the sample are absent from school on at least 5 days during winter because of ill health. [4]

(c) Justify the approximation made in part (b). [1]

X3.12 The ratio of adult males to adult females living in a certain town is 17 : 18, and 2/9 of these adults, independent of gender, do not have a driving license.

(a) Show that the probability that a randomly selected adult in this town is male and has a driving license is equal to 17/45. [1]

(b) Find the probability that, in a randomly selected sample of 25 adults from this town, from 8 to 10 inclusive are females who have a driving license. [4]

X3.13 A fair eight-sided die is numbered 2, 2, 3, 3, 3, 4, 5 and 6. The die is rolled up to and including the roll on which the first 2 is obtained. Let X represent the number of times the die is rolled.

(a) Find E(X). [1]

(b) Show that $P(X \geq 4) = 27/64$. [1]

(c) The die is rolled up to and including the roll on which the first 2 is obtained on 20 occasions. Find, by use of a suitable approximation, the probability that $X \geq 4$ on at least half of these 20 occasions. [4]

(d) Fully justify the approximation used in part (c). [1]

X3.14 A student wishes to approximate the distribution of $X \sim B(240, p)$ by a continuous random variable Y that has a normal distribution.

(a) Find the values of p for which:

(i) approximating X by Y can be justified [3]

(ii) $\text{Var}(Y) < 45$. [3]

(b) Find the range of values of p for which both the approximation is justified and Var(Y) < 45. [2]

Check yourself

C.1 A continuous random variable X ~ N(45, 36). Calculate P(39 ≤ X ≤ 54). [3]

C.2 The variable Y is normally distributed with mean 60 and unknown standard deviation σ. Given that P(Y > 68.4) = 0.10, find the value of σ. [3]

C.3 A normally distributed random variable W ~ N(μ, σ^2) satisfies P(W < 24) = 0.1587 and P(W > 36) = 0.0228. Find the values of μ and σ. [4]

C.4 A biased die has probability 0.2 of showing a six. The die is rolled 150 times.

(a) Explain why a normal approximation to the number of sixes is justified. [1]

(b) Using a suitable continuity correction, find the probability that at least 25 but at most 35 sixes are obtained. [4]

C.5 The mass of packets of flour is normally distributed with mean 1020 g and standard deviation 12 g. Packets with mass under 1000 g are rejected. A sample of 8 packets is selected at random. Calculate the probability that exactly one packet is rejected. [4]

Chapter quiz

Q1 What is the total area under any normal probability density function curve?

A	B	C	D
0.5	1	2π	$\sigma\sqrt{(2\pi)}$

Q2 For the standard normal variable $Z \sim N(\mu, \sigma^2)$, what are the values of μ and σ^2?

A	B	C	D
$\mu = 1, \sigma^2 = 0$	$\mu = 0, \sigma^2 = 1$	$\mu = 0, \sigma^2 = 0$	$\mu = 1, \sigma^2 = 1$

Q3 Which identity correctly relates $\Phi(-z)$ to $\Phi(z)$ for the standard normal distribution?

A	B	C	D
$\Phi(-z) = -\Phi(z)$	$\Phi(-z) = 1 - \Phi(z)$	$\Phi(-z) = 1 + \Phi(z)$	$\Phi(-z) = \Phi(1 - z)$

Q4 If $X \sim N(\mu, \sigma^2)$, which formula transforms X into the standard normal variable Z?

A	B	C	D
$Z = (X - \mu) / \sigma$	$Z = (X - \mu) / \sigma^2$	$Z = (X - \sigma) / \mu$	$Z = (X + \mu) / \sigma$

Q5 If $X \sim N(50, 16)$, what is the exact probability $P(X = 50)$?

A	B	C	D
0.5	0.25	0	1

Q6 Where do the two points of inflection on the normal curve $y = f(x)$ occur?

A	B	C	D
At $x = \mu \pm 0.5$	At $x = \mu \pm \sigma$	At $x = \mu \pm 2\sigma$	At $x = \mu \pm 3\sigma$

Q7 Given $X \sim N(30, 25)$, what is the standardised z-score corresponding to $x = 40$?

A	B	C	D
0.4	1.0	2.0	4.0

Q8 If $\Phi(1.5) = 0.9332$ for $Z \sim N(0, 1)$, what is $P(Z > 1.5)$?

A	B	C	D
0.0668	0.4332	0.5668	0.9332

Q9 Which conditions must be met to approximate $X \sim B(n, p)$ using a normal distribution?

A	B	C	D
$n > 30$ only	$np > 5$ and $n(1 - p) > 5$	$np < 5$ and $p > 0.5$	$n < 50$ and $np = 10$

Q10 When approximating $B(n, p)$ by $N(\mu, \sigma^2)$, what is the continuity correction for $P(X \geq 20)$?

A	B	C	D
$P(Y \geq 19.0)$	$P(Y \geq 19.5)$	$P(Y \geq 20.0)$	$P(Y \geq 20.5)$

Q11 When approximating discrete X by continuous Y, what continuity correction applies to $P(X < 35)$?

A	B	C	D
$P(Y < 34.5)$	$P(Y < 35.0)$	$P(Y < 35.5)$	$P(Y < 36.0)$

Q12 For $X \sim B(100, 0.36)$, what are the mean μ and variance σ^2 of the approximating normal distribution?

A	B	C	D
$\mu = 36, \sigma^2 = 23.04$	$\mu = 36, \sigma^2 = 4.8$	$\mu = 64, \sigma^2 = 23.04$	$\mu = 100, \sigma^2 = 36$

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/s1-8-the-normal-distribution/