S1·7

The binomial and geometric distributions

CHAPTER ASSIGNMENT · 34 MARKS

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. Give exact probabilities (fractions in simplest form) or decimals to 3 significant figures unless specified otherwise.

Practice questions from *Cambridge International AS & A Level Mathematics: Probability & Statistics 1 Coursebook* (Cambridge University Press, Dean Chalmers), Chapter 7 (The binomial and geometric distributions), Exercises 7A to 7D and End-of-chapter review exercise 7, for classroom use. Where a question needs a diagram, the diagram is described in words or given as a table. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 7A · The binomial distribution

7A.1 The variable X has a binomial distribution with n = 4 and p = 0.2. Find: (a) P(X = 4) · (b) P(X = 0) · (c) P(X = 3) · (d) P(X = 3 or 4).

7A.2 Given that Y ~ B(7, 0.6), find: (a) P(Y = 7) · (b) P(Y = 5) · (c) P(Y ≠ 4) · (d) P(3 < Y < 6).

7A.3 Given that W ~ B(9, 0.32), find: (a) P(W = 5) · (b) P(W ≠ 5) · (c) P(W < 2) · (d) P(0 < W < 9).

7A.4 Given that $V \sim B(8, 2/7)$, find: (a) $P(V = 4)$ · (b) $P(V \geq 7)$ · (c) $P(V \leq 2)$ · (d) $P(3 \leq V \leq 6)$ · (e) $P(V$ is an odd number).

7A.5 Find the probability that each of the following events occur: (a) Exactly five heads are obtained when a fair coin is tossed nine times. · (b) Exactly two 6s are obtained with 11 rolls of a fair die.

7A.6 A man has five packets and each contains three brown sugar cubes and one white sugar cube. He randomly selects one cube from each packet. Find the probability that he selects exactly one brown sugar cube.

7A.7 A driving test is passed by 70% of people at their first attempt. Find the probability that exactly five out of eight randomly selected people pass at their first attempt.

7A.8 Research shows that the owners of 63% of all saloon cars are male. Find the probability that exactly 20 out of 30 randomly selected saloon cars are owned by: (a) males · (b) females.

7A.9 In a particular country, 58% of the adult population is married. Find the probability that exactly 12 out of 20 randomly selected adults are married.

7A.10 A footballer has a 95% chance of scoring each penalty kick that she takes. Find the probability that she: (a) scores from all of her next 10 penalty kicks · (b) fails to score from exactly one of her next seven penalty kicks.

7A.11 On average, 13% of all tomato seeds of a particular variety fail to germinate within 10 days of planting. Find the probability that 34 or 35 out of 40 randomly selected seeds succeed in germinating within 10 days of planting.

7A.12 There is a 15% chance of rain on any particular day during the next 14 days. Find the probability that, during the next 14 days, it rains on: (a) exactly 2 days · (b) at most 2 days.

7A.13 A factory makes electronic circuit boards and, on average, 0.3% of them have a minor fault. Find the probability that a random sample of 200 circuit boards contains: (a) exactly one with a minor fault · (b) fewer than two with a minor fault.

7A.14 There is a 50% chance that a six-year-old child drops an ice cream that they are eating. Ice creams are given to 5 six-year-old children. (a) Find the probability that exactly one ice cream is dropped. · (b) 45 six-year-old children are divided into nine groups of five and each child is given an ice cream. Calculate the probability that exactly one of the children in at most one of the groups drops their ice cream.

7A.15 A coin is biased such that heads is three times as likely as tails on each toss. The coin is tossed 12 times. The variables H and T are, respectively, the number of heads and the number of tails obtained. Find the value of $P(H = 7) / P(T = 7)$.

7A.16 Given that $Q \sim B(n, 0.3)$ and that $P(Q = 0) > 0.1$, find the greatest possible value of n.

7A.17 The variable $T \sim B(n, 0.96)$ and it is given that $P(T = n) > 0.5$. Find the greatest possible value of n.

7A.18 Given that $R \sim B(n, 0.8)$ and that $P(R > n - 1) < 0.006$, find the least possible value of n.

7A.19 The number of damaged eggs, D, in cartons of six eggs have been recorded by an inspector at a packing depot. The following table shows the frequency distribution of some of the numbers of damaged eggs in 150000 boxes:

No. damaged eggs (D)	0	1	2	3	4	5	6
No. cartons (f)	141393	8396	a	b	0	0	0

The distribution of D is to be modelled by $D \sim B(6, p)$. (a) Estimate a suitable value for p, correct to 4 decimal places. · (b) Calculate estimates for the value of a and of b. · (c) Calculate an estimate for the least number of additional cartons that would need to be inspected for there to be at least 8400 cartons containing one damaged egg.

7A.20 The number of months during the 4-month monsoon season (June to September) in which the total rainfall was greater than 5 metres, R, has been recorded at a location in Meghalaya for the past 32 years, and is shown in the following table:

No. months	0	1	2	3	4
No. years (f)	2	8	12	8	2

The distribution of R is to be modelled by $R \sim B(4, p)$. (a) Find the value of p, and state clearly what this value represents. · (b) Give a reason why, in real life, it is unlikely that a binomial distribution could be used to model these data accurately.

7A.21 In a particular country, 90% of both females and males drink tea. Of those who drink tea, 40% of the females and 60% of the males drink it with sugar. Find the probability that in a random selection of two females and two males: (a) all four people drink tea · (b) an equal number of females and males drink tea with sugar.

7A.22 It is estimated that 0.5% of all left-handed people and 0.4% of all right-handed people suffer from some form of colour-blindness. A random sample of 200 left-handed and 300 right-handed people is taken. Find the probability that there is exactly one person in the sample that suffers from colour-blindness.

Exercise 7B · Expectation and variance of the binomial distribution

7B.1 Calculate the expectation, variance and standard deviation of each of the following discrete random variables. Give non-exact answers correct to 3 significant figures: (a) $V \sim B(5, 0.2)$ · (b) $W \sim B(24, 0.55)$ · (c) $X \sim B(365, 0.18)$ · (d) $Y \sim B(20, \sqrt{(0.5)})$.

7B.2 Given that $X \sim B(8, 0.25)$, calculate: (a) $E(X)$ and $Var(X)$ · (b) $P[X = E(X)]$ · (c) $P[X < E(X)]$.

7B.3 Given that $Y \sim B(11, 0.23)$, calculate: (a) $P(Y \neq 3)$ · (b) $P[Y < E(Y)]$.

7B.4 Given that $X \sim B(n, p)$, $E(X) = 20$ and $Var(X) = 12$, find: (a) the value of n and of p · (b) $P(X = 21)$.

7B.5 Given that $G \sim B(n, p)$, $E(G) = 24½$ and $Var(G) = 10\ 5/24$, find: (a) the parameters of the distribution of G · (b) $P(G = 20)$.

7B.6 W has a binomial distribution, where $E(W) = 2.7$ and $Var(W) = 0.27$. Find the values of n and p and use them to draw up the probability distribution table for W.

7B.7 Give a reason why a binomial distribution would not be a suitable model for the distribution of X in each of the following situations: (a) X is the height of the tallest person selected when three people are randomly chosen from a group of 10. · (b) X is the number of girls selected when two children are chosen at random from a group containing one girl and three boys. · (c) X is the number of motorbikes selected when four vehicles are randomly picked from a car park containing 134 cars, 17 buses and nine bicycles.

7B.8 The variable $Q \sim B(n, 1/3)$, and its standard deviation is one-third of its mean. Calculate the non-zero value of n and find $P(5 < Q < 8)$.

7B.9 The random variable $H \sim B(192, p)$, and E(H) is 24 times the standard deviation of H. Calculate the value of p and find the value of k, given that $P(H = 2) = k \times 2^{-379}$.

7B.10 It is estimated that 1.3% of the matches produced at a factory are damaged in some way. A household box contains 462 matches. (a) Calculate the expected number of damaged matches in a household box. · (b) Find the variance of the number of damaged matches and the variance of the number of undamaged matches in a household box. · (c) Show that approximately 10.4% of the household boxes are expected to contain exactly eight damaged matches. · (d) Calculate the probability that at least one from a sample of two household boxes contains exactly eight damaged matches.

7B.11 On average, 8% of the candidates sitting an examination are awarded a merit. Groups of 50 candidates are selected at random. (a) How many candidates in each group are not expected to be awarded a merit? · (b) Calculate the variance of the number of merits in the groups of 50. · (c) Find the probability that: (i) three, four or five candidates in a group of 50 are awarded merits, (ii) three, four or five candidates in both of two groups of 50 are awarded merits.

Exercise 7C · The geometric distribution

7C.1 Given the discrete random variable $X \sim \text{Geo}(0.2)$, find: (a) $P(X = 7)$ · (b) $P(X \neq 5)$ · (c) $P(X > 4)$.

7C.2 Given that $T \sim \text{Geo}(0.32)$, find: (a) $P(T = 3)$ · (b) $P(T \leq 6)$ · (c) $P(T > 7)$.

7C.3 The probability that Mike is shown a yellow card in any football match that he plays is 1/2. Find the probability that Mike is next shown a yellow card: (a) in the third match that he plays · (b) before the fourth match that he plays.

7C.4 On average, Diya concedes one penalty in every six hockey matches that she plays. Find the probability that Diya next concedes a penalty: (a) in the eighth match that she plays · (b) after the fourth match that she plays.

7C.5 The sides of a fair 5-sided spinner are marked 1, 1, 2, 3 and 4. It is spun until the first score of 1 is obtained. Find the probability that it is spun: (a) exactly twice · (b) at most five times · (c) at least eight times.

7C.6 It is known that 80% of the customers at a DIY store own a discount card. Customers queuing at a checkout are asked if they own a discount card. (a) Find the probability that the first customer who owns a discount card is: (i) the third customer asked, (ii) not one of the first four customers asked. · (b) Given that 10% of the customers with discount cards forget to bring them to the store, find the probability that the first customer who owns a discount card and remembered to bring it to the store is the second customer asked.

7C.7 In a manufacturing process, the probability that an item is faulty is 0.07. Items from those produced are selected at random and tested. (a) Find the probability that the first faulty item is: (i) the 12th item tested, (ii) not one of the first 10 items tested, (iii) one of the first eight items tested. · (b) What assumptions have you made about the occurrence of faults in the items so that you can calculate the probabilities in part a?

7C.8 Two independent random variables are X ~ Geo(0.3) and Y ~ Geo(0.7). Find: (a) P(X = 2) · (b) P(Y = 2) · (c) P(X = 1 and Y = 1).

7C.9 On average, 14% of the vehicles being driven along a stretch of road are heavy goods vehicles (HGVs). A girl stands on a footbridge above the road and counts the number of vehicles, up to and including the first HGV that passes. Find the probability that she counts: (a) at most three vehicles · (b) at least five vehicles.

7C.10 The probability that a woman can connect to her home Wi-Fi at each attempt is 0.44. Find the probability that she fails to connect until her fifth attempt.

7C.11 Decide whether or not it would be appropriate to model the distribution of X by a geometric distribution in the following situations. In those cases for which it is not appropriate, give a reason: (a) A bag contains two red sweets and many more green sweets. A child selects a sweet at random and eats it, selects another and eats it, and so on. X is the number of sweets selected and eaten, up to and including the first red sweet. · (b) A monkey sits in front of a laptop with a blank word processing document on its screen. X is the number of keys pressed by the monkey, up to and including the first key pressed that completes a row of three letters that form a meaningful three-letter word. · (c) X is the number of times that a grain of rice is dropped from a height of 2 metres onto a chessboard, up to and including the first time that it comes to rest on a white square. · (d) X is the number of races in which an athlete competes during a year, up to and including the first race that he wins.

7C.12 The random variable T has a geometric distribution and it is given that $P(T = 2) / P(T = 5) = 15.625$. Find $P(T = 3)$.

7C.13 $X \sim Geo(p)$ and $P(X = 2) = 0.2464$. Given that $p < 0.5$, find $P(X > 3)$.

7C.14 Given that $X \sim Geo(p)$ and that $P(X \leq 4) = 2385/2401$, find $P(1 \leq X < 4)$.

7C.15 Two ordinary fair dice are rolled simultaneously. Find the probability of obtaining: (a) the first double on the fourth roll · (b) the first pair of numbers with a sum of more than 10 before the 10th roll.

7C.16 X ~ Geo(0.24) and Y ~ Geo(0.25) are two independent random variables. Find the probability that X + Y = 4.

Exercise 7D · Mode and expectation of the geometric distribution

7D.1 Given that X ~ Geo(0.36), find the exact value of E(X).

7D.2 The random variable Y follows a geometric distribution. Given that P(Y = 1) = 0.2, find E(Y).

7D.3 Given that S ~ Geo(p) and that E(S) = 4½, find P(S = 2).

7D.4 Let T be the number of times that a fair coin is tossed, up to and including the toss on which the first tail is obtained. Find the mode and the mean of T.

7D.5 Let X be the number of times an ordinary fair die is rolled, up to and including the roll on which the first 6 is obtained. Find E(X) and evaluate P[X > E(X)].

7D.6 A biased 4-sided die is numbered 1, 3, 5 and 7. The probability of obtaining each score is proportional to that score. (a) Find the expected number of times that the die will be rolled, up to and including the roll on which the first non-prime number is obtained. · (b) Find the probability that the first prime number is obtained on the third roll of the die.

7D.7 Sylvie and Thierry are members of a choir. The probabilities that they can sing a perfect high C note on each attempt are 4/7 and 5/8, respectively. (a) Who is expected to fail fewer times before singing a high C note for the first time? · (b) Find the probability that both Sylvie and Thierry succeed in singing a high C note on their second attempts.

7D.8 A standard deck of 52 playing cards has an equal number of hearts, spades, clubs and diamonds. A deck is shuffled and a card is randomly selected. Let X be the number of cards selected, up to and including the first diamond. (a) Given that X follows a geometric distribution, describe the way in which the cards are selected, and give the reason for your answer. · (b) Find the probability that: (i) X is equal to E(X), (ii) neither of the first two cards selected is a heart and the first diamond is the third card selected.

7D.9 A study reports that a particular gene in 0.2% of all people is defective. X is the number of randomly selected people, up to and including the first person that has this defective gene. Given that $P(X \leq b) > 0.865$, find E(X) and find the smallest possible value of b.

7D.10 Anouar and Zane play a game in which they take turns at tossing a fair coin. The first person to toss heads is the winner. Anouar tosses the coin first, and the probability that he wins the game is $0.5^1 + 0.5^3 + 0.5^5 + 0.5^7 + \ldots$. (a) Describe the sequence of results represented by the value 0.5^5 in this series. · (b) Find, in a similar form, the probability that Zane wins the game. · (c) Find the probability that Anouar wins the game.

End-of-chapter review exercise 7

R7.1 Given that $X \sim B(n, 1/n)$, find an expression for $P(X = 1)$ in terms of n. [2]

R7.2 A family has booked a long holiday in Skragness, where the probability of rain on any particular day is 0.3. Find the probability that: (a) the first day of rain is on the third day of their holiday [1] · (b) it does not rain for the first 2 weeks of their holiday. [2]

R7.3 One plastic robot is given away free inside each packet of a certain brand of biscuits. There are four colours of plastic robot (red, yellow, blue and green) and each colour is equally likely to occur. Nick buys some packets of these biscuits. Find the probability that: (i) he gets a green robot on opening his first packet, [1] · (ii) he gets his first green robot on opening his fifth packet. [2] Nick's friend Amos is also collecting robots. (iii) Find the probability that the first four packets Amos opens all contain different coloured robots. [3] *(Cambridge International AS & A Level Mathematics 9709 Paper 62 Q3 November 2015)*

R7.4 Weiqi has two fair triangular spinners. The sides of one spinner are labelled 1, 2, 3, and the sides of the other are labelled 2, 3, 4. Weiqi spins them simultaneously and notes the two numbers on which they come to rest. (a) Find the probability that these two numbers differ by 1. [2] · (b) Weiqi spins both spinners simultaneously on 15 occasions. Find the probability that the numbers on which they come to rest do not differ by 1 on exactly eight or nine of the 15 occasions. [3]

R7.5 A computer generates random numbers using any of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The numbers appear on the screen in blocks of five digits, such as 50119 26317 40068 ..... Find the probability that: (a) there are no 7s in the first block [1] · (b) the first zero appears in the first block [1] · (c) the first 9 appears in the second block. [2]

R7.6 Four ordinary fair dice are rolled. (a) In how many ways can the four numbers obtained have a sum of 22? [2] · (b) Find the probability that the four numbers obtained have a sum of 22. [2] · (c) The four dice are rolled on eight occasions. Find the probability that the four numbers obtained have a sum of 22 on at least two of these occasions. [3]

R7.7 When a certain driver parks their car in the evenings, they are equally likely to remember or to forget to switch off the headlights. Giving your answers in their simplest index form, find the probability that on the next 16 occasions that they park their car in the evening, they forget to switch off the headlights: (a) 14 more times than they remember to switch them off [2] · (b) at least 12 more times than they remember to switch them off. [3]

R7.8 Gina has been observing students at a university. Her data indicate that 60% of the males and 70% of the females are wearing earphones at any given time. She decides to interview randomly selected students and to interview males and females alternately. (a) Use Gina's observation data to find the probability that the first person not wearing earphones is the third male interviewed, given that she first interviews: (i) a male [2], (ii) a female [2], (iii) a male who is wearing earphones. [2] · (b) State any assumptions made about the wearing of earphones in your calculations for part a. [1]

R7.9 In Restaurant Bijoux 13% of customers rated the food as 'poor', 22% of customers rated the food as 'satisfactory' and 65% rated it as 'good'. A random sample of 12 customers who went for a meal at Restaurant Bijoux was taken. (i) Find the probability that more than 2 and fewer than 12 of them rated the food as 'good'. [3] On a separate occasion, a random sample of n customers who went for a meal at the restaurant was taken. (ii) Find the smallest value of n for which the probability that at least 1 person will rate the food as 'poor' is greater than 0.95. [3] *(Cambridge International AS & A Level Mathematics 9709 Paper 62 Q3 June 2012)*

R7.10 A biased coin is four times as likely to land heads up compared with tails up. The coin is tossed k times so that the probability that it lands tails up on at least one occasion is greater than 99%. Find the least possible value of k. [4]

R7.11 Given that $X \sim B(n, 0.4)$ and that $P(X = 1) = k \times P(X = n - 1)$, express the constant k in terms of n, and find the smallest value of n for which $k > 25$. [5]

R7.12 A book publisher has noted that, on average, one page in eight contains at least one spelling error, one page in five contains at least one punctuation error, and that these errors occur independently and at random. The publisher checks 480 randomly selected pages from various books for errors. (a) How many pages are expected to contain at least one of both types of error? [2] · (b) Find the probability that: (i) the first spelling error occurs after the 10th page [2], (ii) the first punctuation error occurs before the 10th page [2], (iii) the 10th page is the first to contain both types of error. [2]

R7.13 Robert uses his calculator to generate 5 random integers between 1 and 9 inclusive. (i) Find the probability that at least 2 of the 5 integers are less than or equal to 4. [3] Robert now generates n random integers between 1 and 9 inclusive. The random variable X is the number of these n integers which are less than or equal to a certain integer k between 1 and 9 inclusive. It is given that the mean of X is 96 and the variance of X is 32. (ii) Find the values of n and k. [4] *(Cambridge International AS & A Level Mathematics 9709 Paper 62 Q4 June 2013)*

R7.14 Anna, Bel and Chai take turns, in that order, at rolling an ordinary fair die. The first person to roll a 6 wins the game. Find the ratio P(Anna wins) : P(Bel wins) : P(Chai wins), giving your answer in its simplest form. [7]

Check yourself

C7.1 An unbiased 10-sided die numbered 1 to 10 is rolled 15 times. Find the probability of obtaining a prime number on at least 3 but fewer than 6 rolls.

C7.2 The random variable X has a binomial distribution B(n, p). Given that E(X) = 28 and Var(X) = 8.4, calculate n and p, and evaluate P(X = 26).

C7.3 The probability that a customer uses contactless card payment at a coffee shop is 0.72. Find the probability that the first customer who does not use contactless payment is the 5th customer served.

C7.4 A machine produces components of which 4% are defective. If components are inspected one by one, find the probability that more than 30 components must be inspected to find the first defective one.

C7.5 A game ends when the first 6 is rolled with a fair die. Let T be the number of rolls up to and including the first 6. Verify that $E(T) = 6$ and calculate $P(T \leq E(T))$.

Chapter quiz

Q1 Which of the following is NOT a required condition for a binomial distribution B(n, p)?

A	B	C	D
The number of trials n is fixed and finite	The trials are independent of each other	The probability of success changes each trial	There are exactly two possible outcomes

Q2 Given $X \sim B(5, 0.4)$, what is the exact probability $P(X = 2)$?

A	B	C	D
0.2304	0.3456	0.2592	0.1382

Q3 For $X \sim B(4, 0.2)$, what is the probability $P(X \geq 1)$?

A	B	C	D
0.4096	0.5904	0.8000	0.6400

Q4 If $X \sim B(n, 0.3)$ and $P(X = 0) < 0.1$, what is the least possible integer value of n?

A	B	C	D
5	6	7	8

Q5 A variable X ~ B(60, 0.25). What are the mean E(X) and variance Var(X)?

A	B	C	D
E(X) = 15, Var(X) = 11.25	E(X) = 15, Var(X) = 3.75	E(X) = 12, Var(X) = 9.00	E(X) = 20, Var(X) = 15.00

Q6 For X ~ B(n, p), given E(X) = 18 and Var(X) = 7.2, find the values of n and p.

A	B	C	D
n = 30, p = 0.6	n = 45, p = 0.4	n = 25, p = 0.72	n = 36, p = 0.5

Q7 What is the most likely value (mode) of any geometric distribution Geo(p)?

A	B	C	D
Always 1	Always 1/p	0	Depends on the value of p

Q8 Given X ~ Geo(0.2), what is the probability that the first success is on trial 4?

A	B	C	D
0.1024	0.08192	0.0256	0.1280

Q9 If X ~ Geo(0.25), what is the probability P(X > 3)?

A	B	C	D
$0.75^3 = 0.421875$	$1 - 0.75^3 = 0.578125$	$0.75^4 = 0.316406$	$0.25^3 = 0.015625$

Q10 For X ~ Geo(p), which expression represents the cumulative probability P(X ≤ r)?

A	B	C	D
$1 - q^r$	q^r	$1 - p^r$	p^r

Q11 A fair 6-sided die is rolled until the first 6 appears. What is the expected number of rolls?

A	B	C	D
6	5	3.5	1

Q12 Given X ~ Geo(p) and P(X ≤ 2) = 0.64, find the value of p.

A	B	C	D
0.4	0.6	0.36	0.8

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/s1-7-the-binomial-and-geometric-distributions/