S1·6

Probability distributions

CHAPTER ASSIGNMENT

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. Give answers to 3 significant figures unless otherwise specified. Fractions are written a/b, with brackets where needed.

Practice questions from *Cambridge International AS & A Level Mathematics: Probability & Statistics 1 Coursebook* (Cambridge University Press, Dean Chalmers), Chapter 6 (Probability distributions), Exercises 6A, 6B and End-of-chapter review exercise 6, for classroom use. Where a question needs a diagram, the diagram is described in words or given as a table. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 6A · Discrete random variables and probability distributions

6A.1 The discrete random variable V is such that $V \in \{1, 2, 3\}$. Given that $P(V = 1) = P(V = 2) = 2 \times P(V = 3)$, draw up the probability distribution table for V.

6A.2 The probability distribution for the random variable X is given in the following table:

x	2	3	4	5
P(X = x)	p	2p	(1/2)p	3p

Find the value of p and work out $P(2 < X < 5)$.

6A.3 The probability distribution for the random variable W is given in the following table:

w	3	6	9	12	15
P(W = w)	2k	k^2	k/2	4/5 − 3k	13/50

(a) Form an equation using k, then solve it. · (b) Explain why only one of your solutions is valid. · (c) Find P(6 ≤ W < 10).

6A.4 The probability that a boy succeeds with each basketball shot is 7/9. He takes two shots and the discrete random variable S represents the number of successful shots. Show that P(S = 0) = 4/81 and draw up the probability distribution table for S.

6A.5 At a garden centre, there is a display of roses: 25 are red, 20 are white, 15 are pink and 5 are orange. Three roses are chosen at random. (a) Show that the probability of selecting three red roses is approximately 0.0527. · (b) Draw up the probability distribution table for the number of red roses selected. · (c) Find the probability that at least one red rose is selected.

6A.6 Three vehicles from a company's six trucks, five vans, three cars and one motorbike are randomly selected and tested for roadworthiness. (a) Show that the probability of selecting three vans is 2/91. · (b) Draw up the probability distribution table for the number of vans selected. · (c) Find the probability that, at most, one van is selected.

6A.7 Five grapes are randomly selected without replacement from a bag containing one red grape and six green grapes. Name and list the possible values of two discrete random variables in this situation. State the relationship between the values of your two variables.

6A.8 A pack of five DVDs contains three movies and two documentaries. Three DVDs are selected and the following table shows the probability distribution for M, the number of movies selected:

m	1	2	3
P(M = m)	0.3	0.6	0.1

Draw up the probability distribution table for D, the number of documentaries selected.

6A.9 In a particular country, 90% of the population is right-handed and 40% of the population has red hair. Two people are randomly selected from the population. Draw up the probability distribution for X, the number of right-handed, red-haired people selected, and state what assumption must be made in order to do this.

6A.10 A fair 4-sided die, numbered 1, 2, 3 and 5, is rolled twice. The random variable X is the sum of the two numbers on which the die comes to rest. (a) Show that P(X = 8) = 1/8. · (b) Draw up the probability distribution table for X, and find P(X > 6).

6A.11 There are eight letters in a post box, and five of them are addressed to Mr Nut. Mr Nut removes four letters at random from the box. (a) Find the probability that none of the selected letters are addressed to Mr Nut. · (b) Draw up the probability distribution table for N, the number of selected letters that are addressed to Mr Nut. · (c) Describe one significant feature of a vertical line graph or bar chart that could be used to represent the probability distribution for N.

6A.12 A discrete random variable Y is such that $Y \in \{8, 9, 10\}$. Given that $P(Y = y) = ky$, find the value of the constant k.

6A.13 Q is a discrete random variable and Q ∈ {3, 4, 5, 6}. (a) Given that $P(Q = q) = cq^2$, find the value of the constant c. · (b) Hence, find P(Q > 4).

6A.14 Four books are randomly selected from a box containing 10 novels, 10 reference books and 5 dictionaries. The random variable N represents the number of novels selected. (a) Find the value of P(N = 2), correct to 3 significant figures. · (b) Without further calculation, state which of N = 0 or N = 4 is more likely. Explain the reasons for your answer.

6A.15 In a game, a fair 4-sided spinner with edges labelled 0, 1, 2 and 3 is spun. If a player spins 1, 2 or 3, then that is their score. If a player scores 0, then they spin a fair triangular spinner with edges labelled 0, 1 and 2, and the number they spin is their score. Let the variable X represent a player's score. (a) Show that P(X = 0) = 1/12. · (b) Draw up the probability distribution table for X, and find the probability that X is a prime number.

6A.16 A biased coin is tossed three times. The probability distribution for H, the number of heads obtained, is shown in the following table:

h	0	1	2	3
P(H = h)	0.512	0.384	0.096	a

(a) Find the probability of obtaining a head each time the coin is tossed. · (b) Give another discrete random variable that is related to these trials, and calculate the probability that its value is greater than the value of H.

6A.17 Two ordinary fair dice are rolled. A score of 3 points is awarded if exactly one die shows an odd number and there is also a difference of 1 between the two numbers obtained. A player who rolls two even numbers is awarded a score of 2 points, otherwise a player scores 1 point. (a) Draw up the probability distribution table for S, the number of points awarded. · (b) Find the probability that a player scores 3 points, given that the sum of the numbers on their two dice is greater than 9.

6A.18 The discrete random variable R is such that $R \in \{1, 3, 5, 7\}$. (a) Given that $P(R = r) = [k(r + 1)] / (r + 2)$, find the value of the constant k. · (b) Hence, find $P(R \leq 4)$.

Exercise 6B · Expectation and variance of a discrete random variable

6B.1 The probability distribution for the random variable X is given in the following table:

x	0	1	2	3
P(X = x)	0.10	0.12	0.36	0.42

Calculate E(X) and Var(X).

6B.2 The probability distribution for the random variable Y is given in the following table:

y	0	1	2	3	4
P(Y = y)	0.03	2p	0.32	p	0.05

(a) Find the value of p. · (b) Calculate E(Y) and the standard deviation of Y.

6B.3 The random variable T is such that $T \in \{1, 3, 6, 10\}$. Given that the four possible values of T are equiprobable, find E(T) and Var(T).

6B.4 The following table shows the probability distribution for the random variable V:

v	1	3	9	m
P(V = v)	0.4	0.28	0.14	0.18

Given that E(V) = 5.38, find the value of m and calculate Var(V).

6B.5 R is a random variable such that $R \in \{10, 20, 70, 100\}$. Given that $P(R = r)$ is proportional to r, show that $E(R) = 77$ and find $Var(R)$.

6B.6 The probability distribution for the random variable W is given in the following table:

w	2	7	a	24
P(W = w)	0.3	0.3	0.1	0.3

Given that $E(W) = a$, find a and evaluate $Var(W)$.

6B.7 The possible outcomes from a business venture are graded from 5 to 1, as shown in the following table:

Grade	5	4	3	2	1
Outcome	High profit	Fair profit	No loss	Small loss	Heavy loss
Probability	0.24	0.33	0.24	0.11	0.08

(a) Calculate the expected grade and use it to describe the expected outcome of the venture. Find the standard deviation and explain what it gives a measure of in this case. · (b) Investigate the expected outcome and the standard deviation when the grading is reversed (i.e. high profit is graded 1, and so on). Compare these outcomes with those from part a.

6B.8 Two ordinary fair dice are rolled. The discrete random variable X is the lowest common multiple of the two numbers rolled. (a) Draw up the probability distribution table for X. · (b) Find E(X) and P[X > E(X)]. · (c) Calculate Var(X).

6B.9 In a game, a player attempts to hit a target by throwing three darts. With each throw, a player has a 30% chance of hitting the target. (a) Draw up the probability distribution table for H, the number of times the target is hit in a game. · (b) How many times is the target expected to be hit in 1000 games?

6B.10 Two students are randomly selected from a class of 12 girls and 18 boys. (a) Find the expected number of girls and the expected number of boys. · (b) Write the ratio of the expected number of girls to the expected number of boys in simplified form. What do you notice about this ratio? · (c) Calculate the variance of the number of girls selected.

6B.11 A sewing basket contains eight reels of cotton: four are green, three are red and one is yellow. Three reels of cotton are randomly selected from the basket. (a) Show that the expected number of yellow cotton reels is 0.375. · (b) Find the expected number of red cotton reels. · (c) Hence, state the expected number of green cotton reels.

6B.12 A company offers a $1000 cash loan to anyone earning a monthly salary of at least $2000. To secure the loan, the borrower signs a contract with a promise to repay the $1000 plus a fixed fee before 3 months have elapsed. Failure to do this gives the company a legal right to take $1540 from the borrower's next salary before returning any amount that has been repaid. From past experience, the company predicts that 70% of borrowers succeed in repaying the loan plus the fixed fee before 3 months have elapsed. (a) Calculate the fixed fee that ensures the company an expected 40% profit from each $1000 loan. · (b) Assuming that the company charges the fee found in part a, how would it be possible, without changing the loan conditions, for the company's expected profit from each $1000 loan to be greater than 40%?

6B.13 When a scout group of 8 juniors and 12 seniors meets on a Monday evening, one scout is randomly selected to hoist a flag. Let the variable X represent the number of juniors selected over n consecutive Monday evenings. (a) By drawing up the probability distribution table for X, or otherwise, show that E(X) = 1.2 when n = 3. · (b) Find the number of Monday evenings over which 14 juniors are expected to be selected to hoist the flag.

6B.14 An ordinary fair die is rolled. If the die shows an odd number then S, the score awarded, is equal to that number. If the die shows an even number, then the die is rolled again. If on the second roll it shows an odd number, then that is the score awarded. If the die shows an even number on the second roll, the score awarded is equal to half of that even number. (a) List the possible values of S and draw up a probability distribution table. · (b) Find P[S > E(S)]. · (c) Calculate the exact value of Var(S).

6B.15 A fair 4-sided spinner with sides labelled A, B, B, B is spun four times. (a) Show that there are six equally likely ways to obtain exactly two Bs with the four spins. · (b) By drawing up the probability distribution table for X, the number of times the spinner comes to rest on B, find the value of Var(X) / E(X). · (c) What, in the context of this question, does the value found in part b represent?

End-of-chapter review exercise 6

R6.1 Find the mean and the variance of the discrete random variable X, whose probability distribution is given in the following table:

x	1	2	3	4
P(X = x)	1 − k	2 − 3k	3 − 4k	4 − 6k

R6.2 The following table shows the probability distribution for the random variable Y:

y	1	10	q	101
P(Y = y)	0.2	0.4	0.2	0.2

(a) Given that Var(Y) = 1385.2, show that $q^2 - 61q + 624 = 0$ and solve this equation. · (b) Find the greatest possible value of E(Y).

R6.3 An investment company has produced the following table, which shows the probabilities of various percentage profits on money invested over a period of 3 years:

Profit (%)	1	5	10	15	20	30	40	45	50
Probability	0.05	0.10	0.50	0.20	0.05	0.04	0.03	0.02	0.01

(a) Calculate the expected profit on an investment of $50 000. · (b) A woman considers investing $50 000 with the company, but decides that her money is likely to earn more when invested over the same period in a savings account that pays r% compound interest per annum. Calculate, correct to 2 decimal places, the least possible value of r.

R6.4 A chef wishes to decorate each of four cupcakes with one randomly selected sweet. They choose the sweets at random from eight toffees, three chocolates and one jelly. Find the variance of the number of cupcakes that will be decorated with a chocolate sweet.

R6.5 The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The random variable X is the score when the die is thrown. The probability distribution table for X is given:

x	1	2	3	4	5	6
P(X = x)	p	p	p	p	0.2	0.2

The die is thrown 3 times. Find the probability that the score is at least 4 on at least 1 of the 3 throws. [Adapted from 9709/61/M/J/16 Q2]

R6.6 A picnic basket contains five jars: one of marmalade, two of peanut butter and two of jam. A boy removes one jar at random from the basket and then his sister takes two jars, both selected at random. (a) Find the probability that the sister selects her jars from a basket that contains: (i) exactly one jar of jam, (ii) exactly two jars of jam. · (b) Draw up the probability distribution table for J, the number of jars of jam selected by the sister, and show that E(J) = 0.8.

R6.7 Two ordinary fair dice are rolled. The product and the sum of the two numbers obtained are calculated. The score awarded, S, is equal to the absolute (i.e. non-negative) difference between the product and the sum. For example, if 5 and 3 are rolled, then S = (5 × 3) − (5 + 3) = 7. (a) State the value of S when 1 and 4 are rolled. · (b) Draw up a table showing the probability distribution for the 14 possible values of S, and use it to calculate E(S).

R6.8 A fair triangular spinner has sides labelled 0, 1 and 2, and another fair triangular spinner has sides labelled −1, 0 and 1. The score, X, is equal to the sum of the squares of the two numbers on which the spinners come to rest. (a) List the five possible values of X. · (b) Draw up the probability distribution table for X. · (c) Given that X < 4, find the probability that a score of 1 is obtained with at least one of the spinners. · (d) Find the exact value of a, such that the standard deviation of X is (1/a) × E(X).

R6.9 A discrete random variable X, where $X \in \{2, 3, 4, 5\}$, is such that $P(X = x) = (b - x)^2 / 30$. (a) Calculate the two possible values of b. · (b) Hence, find $P(2 < X < 5)$.

R6.10 Set A consists of the ten digits 0, 0, 0, 0, 0, 0, 2, 2, 2, 4. Set B consists of the seven digits 0, 0, 0, 0, 2, 2, 2. One digit is chosen at random from each set. The random variable X is defined as the sum of these two digits. (i) Show that $P(X = 2) = 3/7$. · (ii) Tabulate the probability distribution of X. · (iii) Find E(X) and Var(X). · (iv) Given that X = 2, find the probability that the digit chosen from set A was 2. [9709/63/M/J/10 Q5]

R6.11 The discrete random variable Y is such that $Y \in \{4, 5, 8, 14, 17\}$ and $P(Y = y)$ is directly proportional to $1 / (y + 1)$. Find $P(Y > 4)$.

R6.12 X is a discrete random variable and $X \in \{0, 1, 2, 3\}$. Given that $P(X > 1) = 0.24$, $P(0 < X < 3) = 0.5$ and $P(X = 0 \text{ or } 2) = 0.62$, find $P(X \leq 2 \mid X > 0)$.

R6.13 Four students are to be selected at random from a group that consists of seven boys and x girls. The variables B and G are, respectively, the number of boys selected and the number of girls selected. (a) Given that $P(B = 1) = P(B = 2)$, find the value of x. · (b) Given that $G \neq 3$, find the probability that $G = 4$.

R6.14 A box contains 2 green apples and 2 red apples. Apples are taken from the box, one at a time, without replacement. When both red apples have been taken, the process stops. The random variable X is the number of apples which have been taken when the process stops. (i) Show that $P(X = 3) = 1/3$. · (ii) Draw up the probability distribution table for X. · Another box contains 2 yellow peppers and 5 orange peppers. Three peppers are taken from the box without replacement. (iii) Given that at least 2 of the peppers taken from the box are orange, find the probability that all 3 peppers are orange. [9709/63/N/14 Q7]

R6.15 In a particular discrete probability distribution the random variable X takes the value 120/r with probability r/45, where r takes all integer values from 1 to 9 inclusive. (i) Show that P(X = 40) = 1/15. · (ii) Construct the probability distribution table for X. · (iii) Which is the modal value of X? · (iv) Find the probability that X lies between 18 and 100. [9709/62/N/09 Q5]

Check yourself

C.1 A discrete random variable X has probability distribution $P(X = x) = c / [x(x + 1)]$ for $x \in \{1, 2, 3, 4\}$. (a) Find the exact value of the constant c. · (b) Find P(X > 2).

C.2 The discrete random variable W has expectation E(W) = 6 and variance Var(W) = 4. Find: (a) E(2W + 5) · (b) Var(3 − 4W) · (c) $E(W^2)$.

C.3 A bag contains 4 red balls and 6 blue balls. Three balls are chosen at random without replacement. Let R be the number of red balls chosen. (a) Draw up the probability distribution table for R. · (b) Calculate E(R) and Var(R).

C.4 The random variable Y has distribution: P(Y = −2) = 0.2, P(Y = 0) = 0.5, P(Y = 3) = 0.3. (a) Calculate E(Y) and Var(Y). · (b) If Z = 5Y − 2, find E(Z) and Var(Z).

C.5 In a game, a player rolls a fair four-sided die numbered 1, 2, 3, 4. If the result is 1 or 2, the player's score is twice that number. If the result is 3 or 4, the player rolls again and the score is the sum of the two rolls. Draw up the probability distribution table for the score S, and determine whether the expected score is greater than 5.

Chapter quiz

Q1 The discrete random variable X has P(X = 1) = 0.2, P(X = 2) = 0.5, P(X = 3) = k. Find k.

A	B	C	D
0.7	0.3	0.2	0.5

Q2 A discrete random variable X takes values 1, 2, 3 with $P(X = x) = cx$. Find the constant c.

A	B	C	D
1/3	1/5	1/6	1/2

Q3 The distribution of X is $P(X = 0) = 0.1$, $P(X = 1) = 0.4$, $P(X = 2) = 0.5$. Find $E(X)$.

A	B	C	D
1.4	1.0	1.5	1.2

Q4 For a discrete random variable X, which formula correctly gives the variance $Var(X)$?

A	B	C	D
$[E(X)]^2 - E(X^2)$	$E(X^2) - [E(X)]^2$	$E(X^2) - E(X)$	$E(X - \mu)$

Q5 Given $E(X) = 3$ and $E(X^2) = 13$, calculate the standard deviation of X.

A	B	C	D
4	10	$\sqrt{13}$	2

Q6 If $E(X) = 4$, what is the value of $E(3X - 5)$?

A	B	C	D
12	7	2	17

Q7 If $Var(X) = 5$, what is the value of $Var(3X - 4)$?

A	B	C	D
15	11	45	41

Q8 A fair coin is tossed 3 times. Let X be the number of heads. What is $P(X = 2)$?

A	B	C	D
1/8	3/8	1/2	1/4

Q9 A discrete random variable Y has Var(Y) = 9. What is the standard deviation of 4 − 2Y?

A	B	C	D
6	18	36	12

Q10 The table has P(X = 1) = 0.3, P(X = 2) = 0.4, P(X = 3) = 0.3. What is $E(X^2)$?

A	B	C	D
4.0	2.0	4.6	3.6

Q11 Why is P(X = x) = c(x − 3) invalid for $X \in \{1, 2, 3\}$ when c > 0?

A	B	C	D
The sum exceeds 1	P(X = 3) is zero	P(X = 1) is negative	Values must be even

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Q12 A discrete random variable X has a symmetric distribution around 5. What is E(X)?

A	B	C	D
0	2.5	10	5

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/s1-6-probability-distributions/