S1·5

Permutations and combinations

CHAPTER ASSIGNMENT · 194 MARKS

Name _____ Class _____ No. _____ Date _____

This review exercise covers material across Chapters 4 and 5: Probability, and Permutations and combinations.

Practice questions from *Cambridge International AS & A Level Mathematics: Probability & Statistics 1 Coursebook* (Cambridge University Press, Dean Chalmers), Chapter 5 (Permutations and combinations), Exercises 5A to 5G, End-of-chapter review exercise 5, and Cross-topic review exercise 2 (covering Chapters 4 and 5), for classroom use. Where a question needs a diagram, the diagram is described in words or given as a table. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 5A · The factorial function

5A.1 Without using a calculator, find the value of:

(a) 5! / 3!

(b) 4! / 2! − 3!

(c) 7 × 4! + 21 × 3!

(d) 10! / 8! + 9! / 7!

(e) 20! / 18! − 13! / 11!

5A.2 Use your calculator to find the smallest value of n for which:

(a) $n! > 1\,000\,000$

(b) $5! \times 6! < n!$

(c) $(n!)! > 10^{20}$

5A.3 Use your calculator to find the largest value of n for which:

(a) $n! / 500\,000 < 80$

(b) $1.5 \times 10^{12} - n! > 0$

(c) $n! / (n - 2)! < 500$

5A.4 Express, in as many different ways as possible, the numbers 144, 252 and 1 1/2 in the form $(a! \times b!) / c!$, where none of a, b or c is equal to 0 or to 1.

5A.5 Express the area of a 53 cm by 52 cm rectangle using factorials.

5A.6 Two cubical boxes measure 25 cm by 24 cm by 23 cm, and 8 cm by 7 cm by 6 cm. Express the difference between their volumes using factorials.

5A.7 Eight children each have seven boxes of six eggs and each egg is worth \$0.09. Write the total value of all these eggs in dollars, using factorials.

Exercise 5B · Permutations of n distinct objects

5B.1 In how many ways can the six letters A, B, C, D, E and F be arranged in a row?

5B.2 From a standard deck of 52 playing cards, find how many ways there are of arranging in a row:

(a) all 52 cards

(b) the four kings

(c) the 13 diamonds.

5B.3 In how many different ways can the following stand in a line?

(a) two women

(b) six men

(c) eight adults.

5B.4 In how many different ways can the following sit in a row on a bench?

(a) four girls

(b) three boys

(c) four girls and three boys.

5B.5 Seven cars and x vans can be parked in a line in 39 916 800 ways. Find the number of ways in which five cars and x + 2 vans can be parked in a line.

5B.6 A woman has 10 children. She arranges 11 chairs in a row and sits on the chair in the middle. If her youngest child sits on the adjacent chair to her left, in how many ways can the remaining children be seated?

5B.7 A group of n boys can be arranged in a line in a certain number of ways. By adding two more boys to the group, the number of possible arrangements increases by a factor of 420. Find the value of n.

Exercise 5C · Permutations of n objects with repetitions

5C.1 Find the number of distinct arrangements of all the letters in these words:

(a) TABLE

(b) TABLET

(c) COMMITTEE

(d) MISSISSIPPI

(e) HULLABALLOO.

5C.2 Find how many six-digit numbers can be made from these sets of digits:

(a) 1, 1, 1, 1, 1 and 3

(b) 2, 2, 2, 7, 7 and 7

(c) 5, 6, 6, 6, 7 and 7

(d) 8, 8, 9, 9, 9 and 9.

5C.3 A girl has 20 plastic squares. There are five identical red squares, seven identical blue squares and eight identical green squares. By placing them in a row, joined edge-to-edge, find how many different arrangements she can make using:

(a) one square of each colour

(b) the five red squares only

(c) all of the blue and green squares

(d) all of the 20 squares.

5C.4 Two students are asked to find how many ways there are to plant two trees and three bushes in a row. The first student gives 5! = 120, and the second gives 5! / (2! × 3!) = 10. Decide who you agree with and explain the error made by the other student.

5C.5 Ten coins are placed in a row on a table, each showing a head or a tail.

(a) How many different arrangements of heads and/or tails are possible?

(b) Of the arrangements in part a, find how many have:

(i) five heads and five tails showing

(ii) more heads than tails showing.

5C.6 There are 420 possible arrangements of all the letters in a particular seven-letter word. Give a description of the letters in this word.

5C.7 Find the number of distinct five-letter arrangements that can be made from:

(a) two As and three Bs

(b) two identical vowels and three Bs

(c) two identical vowels and any three identical consonants.

Exercise 5D · Permutations of n distinct objects with restrictions

5D.1 Find how many five-digit numbers can be made using the digits 2, 3, 4, 5 and 6 once each if:

(a) there are no restrictions

(b) the five-digit number must be:

(i) odd

(ii) even

(iii) odd and less than 40 000.

5D.2 Find how many ways four men and two women can stand in a line if:

(a) the two women must be at the front

(b) there must be a woman at the front and a man at the back

(c) the two women must be separated

(d) the four men must not be separated

(e) no two men may stand next to each other.

5D.3 Find the ratio of odd-to-even six-digit numbers that can be made using the digits 1, 2, 3, 4, 5 and 7.

5D.4 Find how many ways 10 books can be arranged in a row on a shelf if:

(a) the two oldest books must be in the middle two positions

(b) the three newest books must not be separated.

5D.5 Five cows and one set of twin calves can be housed separately in a row of seven stalls in ${}^{7}P_{7} = 5040$ ways. Find in how many of these arrangements:

(a) the two calves are not in adjacent stalls

(b) the two calves and their mother, who is one of the 5 cows, are in adjacent stalls

(c) each calf is in a stall adjacent to its mother.

5D.6 Find how many of the six-digit numbers that can be made from 1, 2, 2, 3, 3 and 3:

(a) begin with a 2

(b) are not divisible by 2.

5D.7 Find the number of distinct arrangements that can be made from all the letters in the word THEATRE when the arrangement:

(a) begins with two Ts and ends with two Es

(b) has H as its middle letter

(c) ends with the three vowels E, A and E.

5D.8 A row of five post boxes has owners' names beneath: Mr A, Ms B, Mr C, Ms D, Mrs E. Five parcels, one for the owner of each box, have arrived at the post office. If one parcel is randomly placed in each box, find the number of ways in which:

(a) the five parcels can all be placed in the correct boxes

(b) exactly one parcel can be placed in the wrong box

(c) the correct parcels can be placed in Mr A's and one other person's box only

(d) exactly two parcels can be placed in the correct boxes.

5D.9 There are x boys and y girls to be arranged in a line. Find the relationship between x and y if it is not possible to separate all the boys.

Exercise 5E · Permutations of r objects from n objects

5E.1 Find how many permutations there are of:

(a) five from seven distinct objects

(b) four from nine distinct objects.

5E.2 From 12 books, how many ways are there to select and arrange exactly half of them in a row on a shelf?

5E.3 In how many ways can gold, silver and bronze medals be awarded for first, second and third places in a race between 20 athletes? You may assume that no two athletes tie in these positions.

5E.4 (a) Find the number of ways in which Alvaro can paint his back door and his front door in a different colour if he has 14 colours of paint to choose from. · (b) In how many ways could Alvaro do this if he also considered painting them the same colour?

5E.5 Find how many of the arrangements of four letters from A, B, C, D, E and F:

(a) begin with the letter A

(b) contain the letter A.

5E.6 From a group of 10 boys and seven girls, two are to be chosen to act as the hero and the villain in the school play. Find in how many ways this can be done if these two roles are to be played by:

(a) any of the children

(b) two girls or two boys

(c) a boy and a girl.

5E.7 From a set of 10 rings, a jeweller wishes to display seven of them in their shop window. The formation of the display is a V-shape of 7 rings (one at the apex at the top, and three descending down each side). Find the number of possible displays if, from the set of 10:

(a) the ring with the largest diamond must go at the top of the display

(b) the most expensive ring must go at the top with the two least expensive rings adjacent to it.

5E.8 Using each digit not more than once, how many even four-digit numbers can be made from the digits 1, 2, 3, 4, 5, 6 and 7?

5E.9 Find how many three-digit numbers can be made from the digits 0, 1, 2, 3 and 4, used at most once each, if the three-digit number:

(a) must be a multiple of 10

(b) cannot begin with zero.

5E.10 Give an example of a practical situation where the calculation ${}^{n}P_{r} = 120$ might arise.

5E.11 (a) Under what condition is ${}^{n}P_{r} > {}^{n}P_{n-r}$? · (b) Given that ${}^{n}P_{r} \times {}^{n}P_{n-r} = k \times {}^{n}P_{n}$, find an expression for k in terms of n and r.

5E.12 Five playing cards are randomly selected from a standard deck of 52 cards. These five cards are shuffled, and then the top three cards are placed in a row on a table. How many different arrangements of three of the 52 cards are possible?

5E.13 Seven chairs, A to G, are arranged around a table (chairs A and B on the left, C, D, E along the top, and F, G on the right). In how many ways can the chairs be occupied by 7 of a group of 12 people if three particular people are asked to sit on chairs B, D and F, in any order?

5E.14 A minibus has 11 passenger seats: six seats in a row on the sunny side and five seats in a row on the shady side. Find how many ways eight passengers can be arranged in these seats if:

(a) there are no restrictions

(b) one particular passenger refuses to sit on the sunny side

(c) two particular passengers refuse to sit in seats that are either next to each other or one directly in front of the other.

Exercise 5F · Combinations

5F.1 Find the number of ways in which five apples can be selected from:

(a) eight apples

(b) nine apples and 12 oranges.

5F.2 From seven men and eight women, find how many ways there are to select:

(a) four men and five women

(b) three men and six women

(c) at least 13 people.

5F.3 (a) How many different hands of five cards can be dealt from a standard deck of 52 playing cards? · (b) How many of the hands in part a consist of three of the 26 red cards and two of the 26 black cards?

5F.4 (a) From the 26 letters of the English alphabet, find how many ways there are to choose: (i) six different letters, (ii) 20 different letters. · (b) Use your results from part a to find the condition under which ˣC_y = ˣC_z, where x is a positive integer.

5F.5 In a classroom there are four lights, each operated by a switch that has an on and an off position. How many possible lighting arrangements are there in the classroom?

5F.6 From six boys and seven girls, find how many ways there are to select a group of three children that consists of more girls than boys.

5F.7 A bag contains six red fuses, five blue fuses and four yellow fuses. Find how many ways there are to select:

(a) three fuses of different colours

(b) three fuses of the same colour

(c) 10 fuses in exactly two colours

(d) nine fuses in exactly two colours.

5F.8 The activities offered to children at a school camp are: Morning: acting, painting or singing (3 activities); Afternoon: swimming (which can be done at the pool or at the lake), tennis, golf or cricket (5 choices); Evening: night-hike, star-gazing or drumming (3 activities). If children must choose three activities to fill their day, one from each session, how many sets of three activities are there to choose from?

5F.9 Two taxis are hired to take a group of eight friends to the airport. One taxi can carry five passengers and the other can carry three passengers. What information is given in this situation by the fact that ${}^8C_5 = {}^8C_3 = 56$?

5F.10 Ten cars are to be parked in a car park that has 20 parking spaces set out in two rows of 10. Find how many different patterns of unoccupied parking spaces are possible if:

(a) the cars can be parked in any of the 20 spaces

(b) the cars are parked in the same row

(c) the same number of cars are parked in each row

(d) two more cars are parked in one row than in the other.

5F.11 A boy has eight pairs of trousers, seven shirts and six jackets. In how many ways can he dress in trousers, shirt and jacket if he refuses to wear a particular pair of red trousers with a particular red shirt?

5F.12 A girl has 11 objects to arrange on a shelf but there is room for only seven of them. In how many ways can she arrange seven of the objects in a row along the shelf, if her clock must be included?

5F.13 A Mathematics teacher has 10 different posters to pin up in their classroom but there is enough space for only five of them. They have three posters on algebra, two on calculus and five on trigonometry. In how many ways can they choose the five posters to pin up if:

(a) there are no restrictions

(b) they decide not to pin up either of the calculus posters

(c) they decide to pin up at least one poster on each of the three topics algebra, calculus and trigonometry?

5F.14 As discussed at the beginning of this chapter in Explore 5.1 about encrypting letters, it states that there are over 27 million possibilities for the password encrypted as UJSNOL. How many possibilities are there?

5F.15 How many distinct three-digit numbers can be made from 1, 2, 2, 3, 4 and 5, using each at most once?

Exercise 5G · Problem solving with permutations and combinations

5G.1 Two children are selected at random from a group of six boys and four girls. Use combinations to find the probability of selecting:

(a) two boys

(b) two girls

(c) one boy and one girl.

5G.2 Three chocolates are selected at random from a box containing 10 milk chocolates and 15 dark chocolates. Find the probability of selecting exactly:

(a) two dark chocolates

(b) two milk chocolates

(c) two dark chocolates or two milk chocolates.

5G.3 Four bananas are randomly selected from a crate of 17 yellow and 23 green bananas. Find the probability that:

(a) no green bananas are selected

(b) less than half of those selected are green.

5G.4 A curator has 36 paintings and 44 sculptures from which they will randomly select eight items to display in their gallery. Find the probability that the display consists of at least three more paintings than sculptures.

5G.5 Five people are randomly selected from a group of 67 women and 33 men. Find the probability that the selection consists of an odd number of women.

5G.6 In a toolbox there are 25 screwdrivers, 16 drill bits, 38 spanners and 11 chisels. Find the probability that a random selection of four tools contains no chisels.

5G.7 Five clowns each have a red wig and a blue wig, which they are all equally likely to wear at any particular time. Find the probability that, at any particular time:

(a) exactly two clowns are wearing red wigs

(b) more clowns are wearing blue wigs than red wigs.

5G.8 A gardener has nine rose bushes to plant: three have red flowers and six have yellow flowers. If they plant them in a row in random order, find the probability that:

(a) a yellow rose bush is in the middle of the row

(b) the three red rose bushes are not separated

(c) no two red rose bushes are next to each other.

5G.9 A farmer has 50 animals. They have 24 sheep, of which three are male, and they have 26 cattle, of which 20 are female. A veterinary surgeon wishes to test six randomly selected animals. Find the probability that the selection consists of:

(a) equal numbers of cattle and sheep

(b) more females than males.

5G.10 (a) How many distinct arrangements of the letters in the word STATISTICS are there? · (b) Find the probability that a randomly selected arrangement begins with: (i) three Ts, (ii) three identical letters.

5G.11 Three skirts, four blouses and two jackets are hung in random order on a clothes rail. Find the probability that:

(a) the three skirts occupy the middle section of the arrangement

(b) the two jackets are not separated.

5G.12 In a group of 180 people, there are 88 males, nine of whom are left-handed, and there are 85 females who are not left-handed. If six people are selected randomly from the group, find the probability that exactly four of them are left-handed or female.

5G.13 A small library holds 1240 books: 312 of the 478 novels (N) have hard covers (H), and there are 440 books that do not have hard covers. Some of this information is shown in a Venn diagram where n(N only) = a, n(N ∩ H) = 312, n(H only) = c, and n outside = b.

(a) Find the value of a, of b and of c.

(b) A random selection of 25 of these books is to be donated to a charity group. The charity group hopes that at least 22 of the books will be novels or hard covers. Calculate the probability that the charity group gets what they hope for.

5G.14 A netball team of seven players is to be selected at random from five men and 10 women. Given that at least five women are selected for the team, find the probability that exactly two men are selected.

5G.15 Two items are selected at random from a box that contains some tags and some labels. Selecting two tags is five times as likely as selecting two labels. Selecting one tag and one label is six times as likely as selecting two labels. Find the number of tags and the number of labels in the box.

5G.16 A photograph is to be taken of a pasta dish and n pizzas. The items are arranged in a line in random order. Event X is 'the pasta dish is between two pizzas'.

(a) Investigate the value of P(X) for values of n from 2 to 5.

(b) Hence, express the value of P(X') / P(X) in terms of n. Can you justify your answer for any value of $n \geq 2$?

End-of-chapter review exercise 5

R5.1 The word MARMALADE contains four vowels and five consonants. Find the number of possible arrangements of its nine letters if:

(a) there are no restrictions on the order [1]

(b) the arrangement must begin with the four vowels. [2]

R5.2 Five men, four children and two women are asked to stand in a queue at the post office. Find how many ways they can do this if:

(a) the women must be separated [2]

(b) all of the children must be separated from each other. [3]

R5.3 Find the probability that a randomly selected arrangement of all the letters in the word PALLETTE begins and ends with the same letter. [3]

R5.4 Eight-digit mobile phone numbers issued by the Lemon Network all begin with 79.

(a) How many different phone numbers can the network issue? [1]

(b) Find the probability that a randomly selected number issued by this network:

(i) ends with the digits 97 [2]

(ii) reads the same left to right as right to left. [2]

R5.5 There are 12 books on a shelf. Five books are 15 cm tall; four are 20 cm tall and three are 25 cm tall. Find the number of ways that the books can be arranged on the shelf so that none of them is shorter than the book directly to its right. [2]

R5.6 The 11 letters of the word REMEMBRANCE are arranged in a line.

(a) Find the number of different arrangements if there are no restrictions. [1]

(b) Find the number of different arrangements which start and finish with the letter M. [2]

(c) Find the number of different arrangements which do not have all 4 vowels (E, E, A, E) next to each other. [3]

(d) 4 letters from the letters of the word REMEMBRANCE are chosen. Find the number of different selections which contain no Ms and no Rs and at least 2 Es. [3]

R5.7 Find how many ways 15 children can be divided into three groups of five if:

(a) there are no restrictions [2]

(b) two of the children are brothers who must be in the same group. [3]

R5.8 An entertainer has been asked to give a performance consisting of four items. They know three songs, five jokes, two juggling tricks and can play one tune on the mandolin. Find how many different ways there are for them to choose the four items if:

(a) there are no restrictions on their performance [1]

(b) they decide not to sing any songs [2]

(c) they are not allowed to tell more than two jokes. [3]

R5.9 From a group of nine people, five are to be chosen at random to serve on a committee. In how many ways can this be done if two particular people refuse to serve on the committee together? [3]

R5.10 Twenty teams have entered a tournament. In order to reduce the number of teams to eight, they are put into groups of five and the teams in each group play each other twice. The top two teams in each group progress to the next round. From this point on, teams are paired up, playing each other once with the losing team being eliminated. How many games are played during the whole tournament? [3]

R5.11 A bank provides each account holder with a nine-digit card number that is arranged in three blocks (three digits, two digits, four digits: e.g. 447 - 70 - 3536). Find, in index form, the number of card numbers available if:

(a) there are no restrictions on the digits used [1]

(b) none of the three blocks can begin with 0 [2]

(c) the two digits in the second block must not be the same [2]

(d) the three-, two- and four-digit numbers on the card are even, odd and even, respectively. [3]

R5.12 A basket holds nine flowers: two are pink, three are yellow and four are red. Four of these flowers are chosen at random. Find the probability that at least two of them are red. [4]

R5.13 Find the number of ways in which 11 different pieces of fruit can be shared between three boys so that each boy receives an odd number of pieces of fruit. [5]

R5.14 A bakery wishes to display seven of its 14 types of cake in a row in its shop window. There are six types of sponge cake, five types of cheesecake and three types of fruitcake. Find the number of possible displays that can be made if the bakery places:

(a) a sponge cake at each end of the row and includes no fruitcakes in the display [2]

(b) a fruitcake at one end of the row with sponge cakes and cheesecakes placed alternately in the remainder of the row. [4]

R5.15 Five cards, each marked with a different single-digit number from 3 to 7, are randomly placed in a row. Find the probability that the first card in the row is odd and that the three cards in the middle of the row have a sum of 15. [4]

R5.16 Two ordinary fair dice are rolled and the two faces on which they come to rest are hidden by holding the dice together and lifted off the table. The sum of the numbers on the 10 visible faces of the dice is denoted by T.

(a) Find the number of possible values of T, and find the most likely value of T. [4]

(b) Calculate the probability that $T \leq 38$. [3]

R5.17 Three ordinary fair dice are rolled. Find the number of ways in which the number rolled with the first die can exceed the sum of the numbers rolled with the second and third dice. Hence, find the probability that this event does not occur in two successive rolls of the three dice. [6]

R5.18 How many even four-digit numbers can be made from the digits 0, 2, 3, 4, 5 and 7, each used at most once, when the first digit cannot be zero? [4]

R5.19 (a) (i) Find how many numbers there are between 100 and 999 in which all three digits are different. [3] · (ii) Find how many of the numbers in part i are odd numbers greater than 700. [4]

(b) A bunch of flowers consists of a mixture of roses, tulips and daffodils. Tom orders a bunch of 7 flowers from a shop to give to a friend. There must be at least 2 of each type of flower. The shop has 6 roses, 5 tulips and 4 daffodils, all different from each other. Find the number of different bunches of flowers that are possible. [4]

R5.20 Three identical cans of cola, 2 identical cans of green tea and 2 identical cans of orange juice are arranged in a row. Calculate the number of arrangements if:

(a) the first and last cans in the row are the same type of drink [3]

(b) the 3 cans of cola are all next to each other and the 2 cans of green tea are not next to each other. [5]

Cross-topic review exercise 2

CT2.1 Each of the eight players in a chess team plays 12 games against opponents from other teams. The total number of wins, draws and losses for the whole team are denoted by X, Y and Z, respectively.

(a) State the value of X + Y + Z. [1]

(b) Find the least possible value of Z − X, given that Y = 25. [1]

(c) Given that none of the players drew any of their games and that X − Z = 50, find the exact mean number of games won by the players. [2]

CT2.2 Six books are randomly given to two girls so that each receives at least one book.

(a) In how many ways can this be done? [3]

(b) Are both girls more likely to receive an odd number of books or an even number of books? Give a reason for your answer. [2]

CT2.3 The 60 members of a ballroom dance society wish to participate in a competition but the coach that has been hired has seats for only 57 people. In how many ways can 57 members be selected if the society's president and vice president must be included? [2]

CT2.4 Four discs in two colours and in four sizes are placed in any order on either of two sticks.

(a) Find the number of ways in which the four discs can be arranged so that:

(i) they are all on the same stick [2]

(ii) there are two discs on each stick. [2]

(b) In how many ways can the discs be placed if there are no restrictions? [2]

CT2.5 A fair triangular spinner with sides numbered 1, 2 and 3 is spun three times and the numbers that it comes to rest on are written down from left to right to form a three-digit number.

(a) How many possible three-digit numbers are there? [1]

(b) Find the probability that the three-digit number is:

(i) even [1]

(ii) odd and greater than 200. [2]

CT2.6 A book of poetry contains seven poems, three of which are illustrated. In how many different orders can all the poems be read if no two illustrated poems are read one after the other? [3]

CT2.7 Find the number of ways that seven goats and four sheep can sleep in a row if:

(a) all the goats must sleep next to each other [2]

(b) no two sheep may sleep next to each other. [3]

CT2.8 A teacher is looking for 6 pupils to appear in the school play and has decided to select them at random from a group of 11 girls and 13 boys.

(a) Find the number of ways in which the teacher can select the 6 pupils. [1]

(b) Two roles in the play must be played by girls; three roles must be played by boys, but the fool can be played by a girl or by a boy. If the first pupil selected is to play the role of the fool, find the probability that the fool is played by: (i) a particular girl [1], (ii) a boy. [1]

(c) If instead, the pupil who is to play the fool is the last of the six pupils selected, investigate what effect this change in the order of selection has on the probability that the fool is played by: (i) a particular girl [3], (ii) a boy. [3]

CT2.9 A radio presenter has enough time at the end of their show to play five songs. She has 13 songs by four groups to choose from: five songs by The Anvils, four by The Braziers, three by The Chisels and one by The Dustbins. Find the number of ways she can choose five songs to play if she decides:

(a) that there should be no restrictions [1]

(b) to play all three songs by The Chisels [2]

(c) to play at least one song by each of the four groups. [4]

CT2.10 Students enrolling at an A Level college must select three different subjects to study from the six that are available. One subject must be chosen from each of the option groups A, B and C, as shown in the following table:

Group A	Group B	Group C
Physics	Biology	Mathematics
Chemistry	Physics	Biology
History	Mathematics	Computing

(a) One student has chosen to study History and Mathematics. How many subjects do they have to choose from to complete their selection? [1]

(b) How many combinations of three subjects are available to a student who enrols at this college? [2]

CT2.11 Four ordinary fair dice are arranged in a row. Find the number of ways in which this can be done if the four numbers showing on top of the dice:

(a) are all odd [1]

(b) have a sum that is less than 7. [3]

CT2.12 At company V, 12.5% of the employees have a university degree. At company W, 85% of the employees do not have a university degree. There are 112 employees at company V and 120 employees at company W.

(a) One employee is randomly selected. Find the probability that they: (i) work for company V [1], (ii) have a university degree. [2]

(b) Five employees from company W are selected at random. Find the probability that none of them has a university degree. [2]

CT2.13 One hundred qualified drivers are selected at random. Out of these 100 drivers, of the 40 drivers who wear spectacles, 30 passed their driving test at the first attempt. Altogether, 25 of the drivers did not pass at their first attempt.

(a) Show the data given about the drivers in a clearly labelled table or diagram. [3]

(b) Did these drivers pass the test at their first attempt independently of whether or not they wear spectacles? Explain your answer. [3]

CT2.14 A conference hall has 24 overhead lights. Pairs of lights are operated by switches next to the main entrance, and each switch has three numbered settings: 0 (off), 1 (dim), 2 (bright). Find the number of possible lighting arrangements in the hall if:

(a) there are no restrictions [1]

(b) two particular pairs of lights must be on setting 2 [1]

(c) three lights that are not operated by the same switch and five pairs of lights that are operated by the same switch are not working. [2]

CT2.15 Twelve chairs in two colours are arranged in three rows (1, 2, 3) and four columns (A, B, C, D):

Row 1: A1 (Green), B1 (Blue), C1 (Blue), D1 (Blue).

Row 2: A2 (Green), B2 (Green), C2 (Blue), D2 (Blue).

Row 3: A3 (Green), B3 (Green), C3 (Green), D3 (Blue).

(Total: 6 Green chairs and 6 Blue chairs; Column C has two blue chairs, C1 and C2, and one green chair, C3).

Find in how many ways nine people can sit on these chairs if:

(a) the two blue chairs in column C must remain unoccupied [2]

(b) all of the green chairs must be occupied [3]

(c) more blue chairs than green chairs must be occupied [2]

(d) at least one of the chairs in row 2 must remain unoccupied. [5]

CT2.16 In a certain country, vehicle registration plates consist of seven characters: a letter, followed by a three-digit number, followed by three letters (e.g. B 474 PQR).

The first letter cannot be a vowel; the three-digit number cannot begin with 0; and the first of the last three letters cannot be a vowel or any of the letters X, Y or Z.

(a) Find the number of registration plates available. [2]

(b) Find the probability that a randomly selected registration plate is unassigned, given that there are 48.6 million vehicle owners in the country, and that each owns, on average, 1.183 registered vehicles. [3]

CT2.17 Seats for the guests at an awards ceremony are arranged in two rows of eight (top row: 4 on left, 4 on right) and ten (bottom row: 5 on left, 5 on right), divided by a central aisle (total 18 seats: 9 on left of aisle, 9 on right of aisle). Seats are randomly allocated to 18 guests.

(a) Find the probability that two particular guests are allocated seats:

(i) on the same side of the aisle [3]

(ii) in the same row [3]

(iii) on the same side of the aisle and in the same row. [4]

(b) Give a reason why the answer to part a iii is not equal to the product of the answers to part a i and part a ii. [1]

Check yourself

CY.1 Find the number of arrangements of the letters in the word MATHEMATICS that begin and end with the letter M.

CY.2 A committee of 5 is to be selected from 6 men and 7 women. In how many ways can the committee be formed if it must contain at least 3 women and one particular man refuses to serve with one particular woman?

CY.3 Six different history books and four different geography books are placed in a line on a shelf. Find the probability that no two geography books are placed next to each other.

CY.4 Seven counters numbered 1 to 7 are placed in a bag. Three counters are drawn at random without replacement. Find the probability that the sum of the three numbers drawn is odd.

CY.5 How many 4-digit numbers greater than 4000 can be formed from the digits 0, 1, 3, 4, 6, 8 if no digit can be repeated and the number must be divisible by 5?

Chapter quiz

Q1 What is the exact value of 8! / (5! × 3!)?

A	B	C	D
56	336	672	28

Q2 In how many ways can 7 runners finish in 1st, 2nd, and 3rd place?

A	B	C	D
35	5040	210	120

Q3 How many distinct arrangements can be made using all the letters in PARALLEL?

A	B	C	D
6720	3360	40 320	1680

Q4 Five boys and 2 girls stand in a line. How many arrangements have the 2 girls standing together?

A	B	C	D
720	2880	5040	1440

Q5 Five novels and 3 plays are on a shelf. How many arrangements have no two plays adjacent?

A	B	C	D
14 400	7200	2880	40 320

Q6 A committee of 3 men and 2 women is chosen from 6 men and 5 women. How many committees are possible?

A	B	C	D
120	200	462	240

Q7 A team of 4 is chosen from 5 adults and 4 children. How many teams contain at least one child?

A	B	C	D
126	120	121	115

Q8 If ${}^{n}C_{4} = {}^{n}C_{7}$ for a positive integer n, what is the value of n?

A	B	C	D
28	14	7	11

Q9 Four counters are chosen at random from 6 red and 4 blue. What is the probability that all 4 are red?

A	B	C	D
1/14	1/21	3/28	1/42

Q10 All 6 letters of VECTOR are arranged at random in a row. What is the probability the vowels are separated?

A	B	C	D
1/3	2/3	1/2	3/5

Q11 Four different digits from {2, 3, 5, 7, 8} form a number > 5000 that is even. How many numbers can be made?

A	B	C	D
24	36	30	18

Q12 In how many ways can 8 people be split into two groups of 4 to play a doubles tennis match without set courts?

A	B	C	D
70	105	140	35

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/s1-5-permutations-and-combinations/