S1·4

Probability

CHAPTER ASSIGNMENT · 65 MARKS

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. Give exact probabilities (fractions in simplest form) or decimals to 3 significant figures unless specified otherwise.

Practice questions from *Cambridge International AS & A Level Mathematics: Probability & Statistics 1 Coursebook* (Cambridge University Press, Dean Chalmers), Chapter 4 (Probability), Exercises 4A to 4F and End-of-chapter review exercise 4, for classroom use. Where a question needs a diagram, the diagram is described in words or given as a table. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 4A · Experiments, events and outcomes

4A.1 A teacher randomly selects one student from a group of 12 boys and 24 girls. Find the probability that the teacher selects: (a) a particular boy · (b) a girl.

4A.2 United's manager estimates that the team has a 65% chance of winning any particular game and an 85% chance of not drawing any particular game. (a) What are the manager's estimates most likely to be based on? · (b) If the team plays 40 games this season, find the manager's expectation of the number of games the team will lose. · (c) If the team loses one game more than the manager expects this season, explain why this does not necessarily mean that they performed below expectation.

4A.3 Katya randomly picks one of 10 cards: 6 blue cards (one A, two B, three C) and 4 orange cards (one A, one B, two C). Specifically, the cards are: blue A, orange C, blue B, blue C, orange B, orange A, blue C, orange C, blue B, blue C. If she repeats this 40 times, how many times is Katya expected to pick a card that is not blue and does not have a letter B on it?

4A.4 A numbered wheel is divided into eight sectors of equal size, numbered clockwise 1, 3, 4, 5, 3, 2, 3, 4 (one 1, one 2, three 3s, two 4s, one 5). The wheel is spun until it stops with the arrow pointing at one of the numbers. Axel decides to spin the wheel 400 times. (a) Find the number of times the arrow is not expected to point at a 4. · (b) How many more times must Axel spin the wheel so that the expected number of times that the arrow points at a 4 is at least 160?

4A.5 A bag contains black and white counters, and the probability of selecting a black counter is 1/6. (a) What is the smallest possible number of white counters in the bag? · (b) Without replacement, three counters are taken from the bag and they are all black. What is the smallest possible number of white counters in the bag?

4A.6 When a coin is randomly selected from a savings box, each coin has a 98% chance of not being selected. How many coins are in the savings box?

4A.7 A set of data values is 8, 13, 17, 18, 24, 32, 34 and 38. Find the probability that a randomly selected value is more than one standard deviation from the mean.

4A.8 One student is randomly selected from a school that has 837 boys. The probability that a girl is selected is 4/7. Find the probability that a particular boy is selected.

Exercise 4B · Mutually exclusive events and the addition law

4B.1 Find the probability that the number rolled with an ordinary fair die is: (a) a prime number or a 4 · (b) a square number or a multiple of 3 · (c) more than 3 or a factor of 8.

4B.2 A group of 40 students took a test in Economics. In a Venn diagram, 19 boys (B) took the test and seven students failed the test (F): 16 boys passed, 3 boys failed, 4 girls failed, and 17 girls passed (outside B ∪ F). (a) Describe the 21 students who are members of the set B'. · (b) Find the probability that a randomly selected student is a boy or someone who failed the test.

4B.3 Information about all the animals on a farm is given in the table: Male goats = 5, Female goats = 25, Male sheep = 3, Female sheep = 22. (a) Find the probability that a randomly selected animal is: (i) male or a goat, (ii) a sheep or female. · (b) Find a different way of describing each of the two types of animal in part a.

	Male	Female
Goats	5	25
Sheep	3	22

4B.4 Two ordinary fair dice are rolled and three events are: X: the sum of the two numbers rolled is 6; Y: the difference between the two numbers rolled is zero; Z: both of the numbers rolled are even. (a) List the outcomes that are favourable to: (i) X and Y, (ii) X and Z, (iii) Y and Z. · (b) What do your answers to part a tell you about the events X, Y and Z?

4B.5 The letters A, B, B, B, C, D, D and E are written onto eight cards and placed in a bag. Find the probability that the letter on a randomly selected card is: (a) a vowel or in the word DOMAIN · (b) a consonant or in the word DOUBLE.

4B.6 In a group of 25 boys, nine are members of the chess club (C), eight are members of the debating club (D) and 10 are members of neither of these clubs. The Venn diagram shows n(C only) = a, n(C ∩ D) = b, n(D only) = c, and 10 outside. (a) Find the values of a, b and c. · (b) Find the probability that a randomly selected boy is: (i) a member of the chess club or the debating club, (ii) a member of exactly one of these clubs.

4B.7 Forty girls were asked to name the capital of Cuba and of Hungary; 19 knew the capital of Cuba, 20 knew the capital of Hungary and seven knew both. (a) Draw a Venn diagram showing the number of girls who knew each of these capitals. · (b) Find the probability that a randomly selected girl knew: (i) the capital of Cuba but not of Hungary, (ii) just one of these capitals.

4B.8 In a survey on pet ownership, 36% of the participants own a cat, 20% own a hamster but not a cat, and 8% own a hamster and a cat. What percentage of the participants owns neither a hamster nor a cat?

4B.9 A garage repaired 132 vehicles last month. The number of vehicles requiring electrical (E), mechanical (M) and bodywork (B) repairs are: E only = 12, M only = 47, B only = 26, E ∩ M only = 13, E ∩ B only = 11, M ∩ B only = 18, all three (E ∩ M ∩ B) = 5, none outside = 0. Find the probability that a randomly selected vehicle required: (a) mechanical or bodywork repairs · (b) no bodywork repairs · (c) exactly two types of repair.

4B.10 The 100 students at a technical college study at least one of Pure Mathematics (P), Statistics (S) and Mechanics (M). The numbers studying are: P only = 33, P ∩ M only = 17, M only = 32, M ∩ S only = 7, S only = 11 (with P ∩ S = ∅ and none outside). (a) Who does the number 17 in the diagram refer to? · (b) Find the probability that a randomly selected student studies: (i) Pure Mathematics or Mechanics, (ii) exactly two of these subjects. · (c) List the three subjects in ascending order of popularity.

4B.11 Events X and Y are such that P(X) = 0.5, P(Y) = 0.6 and P(X ∩ Y) = 0.2. (a) State, giving a reason, whether events X and Y are mutually exclusive. · (b) Using a Venn diagram, or otherwise, find P(X ∪ Y). · (c) Find the probability that X or Y, but not both, occurs.

4B.12 A, B and C are events where P(A) = 0.3, P(B) = 0.4, P(C) = 0.3, P(A ∩ B) = 0.12, P(A ∩ C) = 0 and P(B ∩ C) = 0.1. (a) State which pair of events from A, B and C is mutually exclusive. · (b) Using a Venn diagram, or otherwise, find P[(A ∪ B ∪ C)'], which is the probability that neither A nor B nor C occurs.

4B.13 A 30 cm square board has two rectangular cards attached: an 8 cm by 12 cm card (area 96 cm²) and a 15 cm by 20 cm card (area 300 cm²). The larger card covers one-quarter of the 8 cm by 12 cm card. A dart is randomly thrown at the 30 cm by 30 cm board (area 900 cm²). Calculate the probability that the dart pierces: (a) both cards · (b) at least one of the cards · (c) exactly one of the cards.

4B.14 Given that P(A) = 0.4, P(B) = 0.7 and that P(A ∪ B) = 0.8, find: (a) P(A ∪ B') · (b) P(A' ∩ B).

4B.15 Each of 27 tourists was asked which of Angola (A), Burundi (B) and Cameroon (C) they had visited: 15 visited A, 8 visited B, 12 visited C, 2 visited all three, 21 visited only one country, 4 of those who visited A visited only one other country, 5 of those who had not visited A visited B only, and all 27 visited at least one country. (a) Draw a fully labelled Venn diagram to illustrate this information. · (b) Find the number of tourists in set B' and describe them. · (c) Describe the tourists in set (A ∪ B) ∩ C' and state how many there are. · (d) Find the probability that a randomly selected tourist had visited at least two of these three countries.

Exercise 4C · Independent events and the multiplication law

4C.1 Using a tree diagram, find the probability that exactly one head is obtained when two fair coins are tossed.

4C.2 Two ordinary fair dice are rolled. Using a possibility diagram, find the probability of obtaining: (a) two 6s · (b) two even numbers · (c) two numbers whose product is 6.

4C.3 It is known that 8% of all new FunX cars develop a mechanical fault within a year and that 15% independently develop an electrical fault within a year. Find the probability that within a year a new FunX car develops: (a) both types of fault · (b) neither type of fault.

4C.4 A certain horse has a 70% chance of winning any particular race. Find the probability that it wins exactly one of its next two races.

4C.5 The probabilities that a team wins, draws or loses any particular game are 0.6, 0.1 and 0.3, respectively. (a) Find the probability that the team wins at least one of its next two games. · (b) If 2 points are awarded for a win, 1 point for a draw and 0 points for a loss, find the probability that the team scores a total of more than 1 point in its next two games.

4C.6 On any particular day, there is a 30% chance of snow in Slushly. Find the probability that it snows there on: (a) none of the next 3 days · (b) exactly one of the next 3 days.

4C.7 Fatima enters three independent sporting events: shot put (chance of winning 85%), javelin (40%), discus (64%). (a) Assuming the events are independent, find the probability that Fatima wins: (i) the shot put and discus, (ii) the shot put and discus only, (iii) exactly two of these events. · (b) What does 'the three events are independent' mean here? Give a reason why this may not be true in real life.

4C.8 A fair six-sided spinner, P, has edges marked 0, 1, 2, 2, 3 and 4. A fair four-sided spinner, Q, has edges marked 0, −1, −1 and −2. Each spinner is spun once and the scores are added to give S. Find: (a) $P(S = 2)$ · (b) $P(S^2 = 1)$.

4C.9 Speedipost couriers delivery times: Letters (40% same day, 50% after 1 day, 10% after 2 days); Packages (15% same day, 55% after 1 day, 30% after 2 days). (a) Is there any truth in the statement 'If you post 10 letters on Monday then only nine of them will be delivered before Wednesday'? Give a reason. · (b) Find the probability that when three letters are posted on Monday, none are delivered on Tuesday. · (c) Find the probability that when a letter and a package are posted together, the letter arrives at least 1 day before the package.

4C.10 A histogram of bus departure delay times has four intervals: 0 to 1 min (frequency density 20 %/min, area 20%), 1 to 3 min (frequency density 15 %/min, area 30%), 3 to 6 min (frequency density 10 %/min, area 30%), 6 to 10 min (frequency density 5 %/min, area 20%). Two buses are selected at random. Calculate an estimate of the probability that: (a) both departures were delayed by less than 4 minutes · (b) at least one of the buses departed more than 7 minutes late.

4C.11 Praveen calls his friend, who answers with constant probability 0.6. If his friend doesn't answer, Praveen calls later, trying up to four times altogether. Find the probability that Praveen speaks with his friend: (a) after making fewer than three calls · (b) on the telephone on this occasion.

4C.12 Each morning, Ruma randomly selects and buys one of four newspapers available at her local shop. Find the probability that she buys: (a) the same newspaper on two consecutive mornings · (b) three different newspapers on three consecutive mornings.

4C.13 A coin is biased such that the probability that three tosses all result in heads is 125/512. Find the probability of obtaining no heads with three tosses of the coin.

4C.14 In a group of five men and four women, there are three pairs of male and female business partners and three teachers, where no teacher is in a business partnership. One man and one woman are selected at random. Find the probability that they are: (a) both teachers · (b) in a business partnership with each other · (c) each in a business partnership but not with each other.

4C.15 A biased five-faced pyramid die has scores 1, 2, 3, 4 and 5 with P(x) = (k − x)/25, where k is a constant. (a) Find, in terms of k, the probability of scoring: (i) 5, (ii) less than 3. · (b) The die is rolled three times and the scores added. Evaluate k and find the probability that the sum of the three scores is less than 5.

4C.16 A game track consists of START followed by squares 1 to 18, where squares 2, 7, 12 and 16 are coloured. Players roll a fair die to advance; ending on a coloured square sends the counter back to START. (a) Find the probability that a player's counter is on 'start' after rolling the die: (i) once, (ii) twice. · (b) Find the probability that after rolling the die three times, a player's counter is on: (i) 18, (ii) 17.

Exercise 4D · Testing for independence

4D.1 Y and Z are independent events with P(Y) = 0.7 and P(Z) = 0.9. Find P(Y ∩ Z).

4D.2 Two independent events are M and N. Given that P(M) = 0.75 and P(M ∩ N) = 0.21, find P(N).

4D.3 Independent events S and T are such that P(S) = 0.4 and P(T') = 0.2. Find: (a) P(S ∩ T) · (b) P(S' ∩ T).

4D.4 A, B and C are independent events, with P(A ∩ B) = 0.35, P(B ∩ C) = 0.56 and P(A ∩ C) = 0.4. (a) Express P(A) in terms of: (i) P(B), (ii) P(C). · (b) Use your answers to part a to find: (i) P(B), (ii) P(A'), (iii) P(B' ∩ C').

4D.5 In a class of 28 children, 19 attend drama classes (D), 13 attend singing lessons (S), and 6 attend both. One child is chosen at random. (a) Illustrate the data in an appropriate table or diagram. · (b) Are events D and S independent? Give a reason for your answer.

4D.6 Each child in a group of 80 was asked whether they regularly read (R) or regularly watch a movie (M). A Venn diagram shows: R only = 12, R ∩ M = 20, M only = 30, outside = 18. Determine, with justification, whether events R and M are independent.

4D.7 Two fair 4-sided dice (faces 1, 2, 3, 4) are rolled. Event A is 'the sum of the numbers is prime'. Event B is 'the product of the numbers is even'. (a) Find, in simplest form, P(A), P(B) and P(A ∩ B). · (b) Determine, with justification, whether events A and B are independent. · (c) Give a reason why events A and B are not mutually exclusive.

4D.8 Two ordinary fair dice are rolled. Event X is 'the product of the two numbers is odd'. Event Y is 'the sum of the two numbers is a multiple of 3'. (a) Determine, giving reasons, whether X and Y are independent. · (b) Are events X and Y mutually exclusive? Justify your answer.

4D.9 A fair 8-sided die (faces 1 to 8) is rolled twice. Event V is 'one score is exactly 4 less than the other'. Event W is 'the product of the scores is less than 13'. Determine whether events V and W are independent, justifying your answer.

4D.10 Two hundred children are categorised by gender and bicycle ownership: 60 of the 108 males own a bicycle; altogether 90 children do not own a bicycle. (a) Tabulate these data. · (b) Determine, giving reasons, whether bicycle ownership is independent of gender. · (c) What percentage of females and what percentage of males own bicycles? Explain how these confirm part b.

4D.11 People voted for Party X independently of income group: Low (3100 voters), Medium (6820 voters), High (2480 voters), Total = 12400 voters. In total 7440 voted for Party X (a from Low, b from Medium, c from High). Find the value of a, of b and of c.

4D.12 Speeds of 207 vehicles at a 120 km/h junction: Under limit (North 36, East 27, South 36, West 39; Total 138); Over limit (North 15, East 15, South 18, West 21; Total 69). Direction totals: North 51, East 42, South 54, West 60. Providing evidence, determine which vehicles' speeds were independent of their direction of travel.

Exercise 4E · Conditional probability

4E.1 One letter is randomly selected from the word BANANA. Find the probability that: (a) an N is selected, given that an A is not selected · (b) an A is selected, given that an N is not selected.

4E.2 In a group of 100 children, 16 have brothers only, 48 have both brothers and sisters, 24 have sisters only, and 12 have neither. Find the probability that a randomly selected child has: (a) sisters, given that they have brothers · (b) brothers, given that they do not have sisters · (c) sisters or brothers, given that they do not have both.

4E.3 Two photographs are randomly selected from 12 colour and eight black-and-white photographs. Find the probability that the second photograph is colour, given that the first is: (a) colour · (b) black and white.

4E.4 Career interests of 40 girls: Nursing (N), Dentistry (D), Human rights (H). Venn diagram shows: N only = 8, N ∩ D only = 3, D only = 11, N ∩ H only = 4, all three = 1, D ∩ H only = 2, H only = 5, none = 6. (a) Find the probability that a girl has an interest in: (i) human rights, given nursing, (ii) nursing, given no interest in dentistry. · (b) Describe any group of girls for whom dentistry is the least popular career of interest.

4E.5 Quiz marks of 40 students: mark 0 (1 student), 1 (1), 2 (2), 3 (4), 4 (4), 5 (8), 6 (6), 7 (5), 8 (5), 9 (3), 10 (1). Two students are selected at random. Find the probability that the second student: (a) scored more than 5, given that the first did not score more than 5 · (b) scored more than 7, given that the first scored more than 7.

4E.6 Task completion times for 115 men: 0 to 1 min (10 men), 1 to 3 min (40 men), 3 to 6 min (45 men), 6 to 10 min (20 men). Two men are selected at random. Find the probability that: (a) the first man took < 1 min, given he took < 3 min · (b) the second took < 6 min, given the first took < 1 min.

4E.7 At an insurance company, 60% of staff are male (M) and 70% work full-time (FT). A Venn diagram shows outside M ∪ FT is 0.10, M only is a, both M and FT is b, FT only is c. (a) What information is given by the value 0.10? · (b) Find the value of a, of b and of c. · (c) An employee is randomly selected. Find: (i) P(M | FT), (ii) P(FT | M'), (iii) P[(M ∩ FT) | (M ∪ FT)].

4E.8 Two fair triangular spinners (sides 1, 2, 3) are spun. Given that the sum of the two numbers is even, find the probability that the two numbers are the same.

4E.9 Two ordinary fair dice are rolled and their scores added. Given that the sum is greater than 6, find the probability that it is not greater than 8.

4E.10 A circular archery target has concentric circles of radii 3 cm (score 5), 9 cm (score 3), 15 cm (score 2) and 30 cm (score 1). (a) Show that the probability of scoring 5 points is 0.01. · (b) Find the probability of scoring 3 points, 2 points and 1 point with an arrow. · (c) Given that an arrow does not score 5 points, find the probability that it scores 1 point. · (d) Given that a total of 6 points is obtained with two arrows, find the probability that neither arrow scores 1 point.

Exercise 4F · Dependent events and conditional probability

4F.1 Two ties are taken at random from a bag of three plain and five striped ties. By use of a tree diagram, or otherwise, find the probability that both ties are: (a) plain · (b) striped.

4F.2 There are four toffee sweets and seven nutty sweets in a girl's pocket. Find the probability that two sweets, selected at random, one after the other, are not the same type.

4F.3 On a library shelf there are seven novels, three dictionaries and two atlases. Two books are randomly selected without replacement. Find the probability that the selected books are: (a) both novels · (b) both dictionaries or both atlases.

4F.4 A woman travels to work by bicycle 70% of the time (late 3% of the time) and by scooter 30% of the time (late 2% of the time). (a) Find the probability that she is late for work on any particular day. · (b) Given that she expects not to be late on approximately 223 days in a year, find the number of days in a year on which she works.

4F.5 Two children are randomly selected from five boys and seven girls. Determine which is more likely to be selected: (a) two boys or two girls? · (b) the two youngest girls or the two oldest boys?

4F.6 A boy has five different pairs of shoes mixed up under his bed. Find the probability that when he selects two shoes at random they can be worn as a matching pair.

4F.7 A bag contains five 4 cm nails, six 7 cm nails and nine 10 cm nails. Find the probability that two randomly selected nails: (a) have a total length of 14 cm · (b) are both 7 cm long, given that they have a total length of 14 cm.

4F.8 Yvonne and Novac play two tennis games every Saturday. Yvonne has a 65% chance of winning the first game; if she wins, her chance of winning the second increases to 70%, but if she loses, her chance decreases to 55%. Find the probability that Yvonne: (a) loses the second game · (b) wins the first game, given that she loses the second game.

4F.9 (a) Given $P(X \cap Y) = 0.13$ and $P(X) = 0.65$, find $P(Y \mid X)$. · (b) Given $P(M \cap N) = 0.27$ and $P(N \mid M) = 0.81$, find $P(M)$. · (c) Given $P(V \cap W) = 0.35$ and $P(W') = 0.60$, find $P(V \mid W)$.

4F.10 When a customer at a furniture store makes a purchase, there is a 15% chance that they purchase a bed. Given that 4.2% of all customers at the store purchase a bed, find the probability that a customer does not make a purchase at the store.

4F.11 A number between 10 and 100 inclusive is selected at random. Find the probability that the number is a multiple of 5, given that none of its digits is a 5.

4F.12 Three of Mr Jumbillo's seven children, who include one set of twins, are selected at random. (a) Calculate the probability that exactly one of the twins is selected. · (b) Given that exactly three of the children are girls, find the probability that the selection of three children contains more girls than boys.

4F.13 Anya calls Zara's mobile with probability 0.8 (Zara answers with probability 0.74) or landline with probability 0.2 (Zara answers with probability y). (a) Given that Zara answers 68% of Anya's calls, find the value of y. · (b) Given that Anya's call is not answered, find the probability that it was made to Zara's landline.

4F.14 Arif offers to take his sons to the beach (probability x) or to the park (probability 1 – x). The sons refuse the beach with probability 0.65 and accept the park with probability 0.85. (a) Find the value of x, given that 33% of Arif's offers are refused. · (b) Given that an offer is accepted, find the probability that it was to the park.

4F.15 Two children are selected from a group in which there are 10 more boys than girls. Given that there are 756 equiprobable ordered selections that can occur, find the probability that two boys or two girls are selected.

4F.16 There is a 43% chance that Riya meets Jasmine when travelling to work. Riya walks to work and does not meet Jasmine 30% of the time, and travels by another method and meets Jasmine 25% of the time. Find the probability that Riya walks to work on any particular day.

4F.17 Aaliyah buys a randomly selected magazine containing a crossword on five randomly chosen days of each week. On 84% of occasions that she buys one, she attempts the crossword, completing it 60% of the time. Find the probability that, on any particular day, Aaliyah does not complete the crossword in a magazine.

End-of-chapter review exercise 4

R4.1 Four quality-control officers tested 1214 randomly selected components from a production line and reported defect proportions: 2/333, 3/411, 0/187 and 4/283. These figures confirm that about k% of components are defective. Of 7150 components produced next month, approximately how many does the manager expect to be defective? [2]

R4.2 Three referees are needed at an international tournament chosen at random from 12: three from Bosnia, four from Chad and five from Denmark. Find the probability that at least two are from the same country. [3]

R4.3 A company employs 117 staff: Male (42 full-time qualified, 10 part-time qualified, 3 full-time unqualified, 7 part-time unqualified), Female (33 full-time qualified, 16 part-time qualified, 2 full-time unqualified, 4 part-time unqualified). Two employees are selected at random. Find the probability that: (a) one is a qualified male and the other an unqualified female [2] · (b) both are unqualified, given that neither is employed part-time. [3]

R4.4 Books read in past 3 months by reading club members: <2 (3), 2–4 (5), 5–7 (22), 8–9 (7), 10 (2), >10 (1). Total = 40. Find the probability that three randomly selected members have all read fewer than eight books, given that they have all read more than four books. [3]

R4.5 Three small devices independently produce voltage outputs of 1, 2 or 3 volts with probabilities 0.3, 0.6 and 0.1, respectively. Find the probability that three of these devices produce an output with a sum of 5 or 6 volts. [4]

R4.6 One hundred attendees at a conference: Male (M, total 47), brown eyes (BE, total 55), right-handed (RH, total 87). Also 39 are right-handed males with brown eyes. (a) Given 43 males have brown eyes, 42 males are right-handed and 46 right-handed attendees have brown eyes, determine the numbers in all eight regions of the Venn diagram. [3] · (b) Two attendees are selected at random. Find the probability that: (i) both are females who are not right-handed [2], (ii) exactly one is right-handed, given neither has brown eyes. [3]

R4.7 A student travels to college by route A (P = 0.3) or route B (P = 0.7). Overall probability of being passed by a bus is 0.034. They are twice as likely to be passed by a bus on route B (prob y) as on route A (prob x). (a) Use a tree diagram to form and solve simultaneous equations in x and y. [3] · (b) Find the probability the student uses route B, given they are not passed by a bus. [3]

R4.8 Two ordinary fair dice are rolled. If the first die shows < 3, the score is the mean of the numbers; otherwise the score is half the absolute difference between the numbers. Find the probability that the score is: (a) positive [3] · (b) greater than 1, given it is less than 2 [3] · (c) less than 2, given it is greater than 1. [3]

R4.9 Three friends independently choose one of four football ground entrances (A, B, C, D). Rick: P(A) = 1/3, P(B) = P(C) = P(D) = 2/9. Brenda: P = 1/4 each. Ali: P(C) = 2/7, P(D) = 3/5, P(A) = P(B) = 2/35. (i) Find the probability that at least 2 friends choose entrance B. [4] · (ii) Find the probability that all three choose the same entrance. [4]

R4.10 Maria chooses toast for breakfast with probability 0.85, else a bread roll. P(jam | toast) = 0.8, P(jam | roll) = 0.4. (i) Draw a fully labelled tree diagram. [2] · (ii) Given Maria did not have jam, find the probability that she had toast. [4]

R4.11 Data on GDP and birth rate for 170 countries: Low GDP (3 Low BR, 5 Med BR, 45 High BR; total 53); Med GDP (20 Low BR, 42 Med BR, 12 High BR; total 74); High GDP (35 Low BR, 8 Med BR, 0 High BR; total 43). Column totals: Low BR 58, Med BR 55, High BR 57. One country chosen at random: (i) Find P(medium GDP). [1] · (ii) Find P(low BR | not medium GDP). [2] · (iii) State whether 'high GDP' and 'high BR' are exclusive, with a reason. [2] · A country is chosen at random from those with medium GDP, and then a different country is chosen from those with medium birth rate. (iv) Find the probability that both countries have medium GDP and medium birth rate. [3]

R4.12 Three boxes contain balls: Box A (2 blue, 3 orange; total 5), Box B (1 blue, 3 orange; total 4), Box C (2 blue, 2 orange; total 4). (a) A box is chosen at random and one ball drawn. Given it is blue, find the probability it came from Box C. [3] · (b) One ball is drawn from each box. Given exactly one blue ball is drawn, find the probability it came from Box A. [4]

R4.13 A and B are independent events with P(A) = 0.45 and P(B) = 0.64. Find P[(A ∪ B)']. [2]

R4.14 Two fair dice are rolled. Event A is 'sum is 2, 3 or 4'. Event B is 'absolute difference is 2, 3 or 4'. (a) When dice show 1 and 3, explain why this shows A and B are not mutually exclusive. [1] · (b) Explain how you know that P(A ∩ B) = 1/18. [2] · (c) Determine whether A and B are independent. [3]

R4.15 A and B are independent events with $P(A \cap B') = 0.14$, $P(A' \cap B) = 0.39$ and $P(A \cap B) < 0.25$. Use a Venn diagram, or otherwise, to find $P[(A \cup B)']$. [4]

R4.16 Adults surveyed about watching evening TV news: Venn diagram of Male vs No shows: Male only = 90, Male ∩ No = x, Female No = 63, Female Yes = 105. Given 'female' and 'watches evening news' are independent, find the total number of adults surveyed. [4]

R4.17 Hotel meal bookings for 71 guests: Breakfast (B), Lunch (L), Supper (S). Venn diagram shows: B only = 24, B ∩ L only = 6, L only = 9, B ∩ S only = 7, all three = 3, L ∩ S only = 5, S only = 16, outside = 1. (a) A guest who has not booked all three meals is selected at random. Find the probability that this guest: (i) booked breakfast or supper [2], (ii) has not booked supper, given that they booked lunch. [2] · (b) Find the probability that two randomly selected guests have both booked lunch, given that they have both booked at least two meals. [3]

R4.18 Three strangers meet on a train. Assuming birthdays are equally likely in any of the 12 months, find the probability that at least two were born in the same month. [4]

R4.19 A box contains three black and four white chess pieces. Five pieces are drawn one at a time without replacement. Find the probability of getting exactly two black pieces which are drawn consecutively. [4]

R4.20 Number of students by IGCSE passes (I = 5, 6, 7, 8, 9, 10) and A Level passes (A = 2, 3, 4, 5):

A Levels \ IGCSEs	5	6	7	8	9	10
2	13	9	5	4	2	0
3	7	8	9	6	1	0
4	0	0	1	0	1	0
5	0	0	0	0	0	1

(a) For a student selected at random, find: (i) $P(I + A = 11 \mid A < 4)$ [2], (ii) $P(I - A > 5 \mid I + A > 10)$. [3] · (b) Six students with at least three A Level passes are selected at random. Find the greatest possible range of the total number of IGCSE passes they could have. [2]

Check yourself

CY4.1 Events G and H satisfy P(G) = 0.55, P(H) = 0.40, and P(G ∪ H) = 0.73. (a) Calculate P(G ∩ H). · (b) Calculate P(G ∩ H'). · (c) Determine, with mathematical justification, whether G and H are independent.

CY4.2 A biased four-sided die has faces marked 1, 2, 3, 4 with probabilities P(1) = 0.1, P(2) = 0.4, P(3) = 0.3, and P(4) = 0.2. The die is rolled twice independently. (a) Find the probability that the product of the two numbers rolled is at least 6. · (b) Given that the sum of the two numbers rolled is odd, find the probability that at least one of the numbers rolled is a 2.

CY4.3 In a college of 120 students, 68 study Physics, 52 study Chemistry, and 24 study neither subject. (a) Draw a Venn diagram to display this information. · (b) Find the probability that a randomly chosen student studies Chemistry, given that they do not study Physics. · (c) Show whether studying Physics and studying Chemistry are independent for these students.

CY4.4 A box contains 6 red balls and 4 blue balls. Two balls are drawn one after another without replacement. (a) Draw a fully labelled tree diagram showing all branch probabilities and path outcomes. · (b) Find the probability that the two balls drawn are of different colours. · (c) Given that at least one red ball is drawn, find the probability that both balls drawn are red.

CY4.5 Factory machine A produces 65% of the total daily output, of which 4% are defective. Machine B produces the remaining 35% of output, of which 2% are defective. An item is selected at random from a day's production. (a) Find the probability that the selected item is defective. · (b) Given that the selected item is defective, find the probability that it was produced by Machine A.

CY4.6 A bag contains 5 green marbles, 3 yellow marbles, and 2 purple marbles. Three marbles are drawn at random without replacement. (a) Find the probability that all three marbles drawn are of different colours. · (b) Find the probability that at least two green marbles are drawn, given that no purple marbles are drawn.

Chapter quiz

Q1 If the probability that a machine develops a fault on any day is 0.08, what is the probability it does not?

A	B	C	D
0.82	0.92	0.02	0.80

Q2 Katya rolls a fair six-sided die 150 times. How many times is she expected to roll a number greater than 4?

A	B	C	D
25	75	50	100

Q3 Events A and B are mutually exclusive. Which of the following statements must be true?

A	B	C	D	
$P(A \cap B) = P(A) \times P(B)$	$P(A \cup B) = 1$	$P(A \cap B) = 0$	P(A	B) = P(A)

Q4 Given $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cup B) = 0.85$, what is $P(A \cap B)$?

A	B	C	D
0.25	0.15	0.30	0.35

Q5 In a group, $P(A) = 0.4$ and $P(A \cup B) = 0.7$. If A and B are mutually exclusive, what is $P(B')$?

A	B	C	D
0.3	0.7	0.6	0.4

Q6 Two independent events X and Y have $P(X) = 0.3$ and $P(Y) = 0.8$. Find $P(X \cap Y)$.

A	B	C	D
0.50	0.56	0.11	0.24

Q7 Given $P(A) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.25$, are A and B independent?

A	B	C	D
Yes, because $P(A \cap B) > 0$	No, because $P(A \cap B) \neq P(A) \times P(B)$	Yes, because $P(A \cup B) \leq 1$	No, because $P(A) + P(B) \neq 1$

Q8 Given $P(A \cap B) = 0.18$ and $P(B) = 0.60$, find $P(A \mid B)$.

A	B	C	D
0.108	0.42	0.30	0.78

Q9 In a class of 30 students, 18 study Art and 8 study both Art and Music. Given a student studies Art, what is P(Music)?

A	B	C	D
8/30	4/9	4/15	10/18

Q10 A bag has 3 red and 5 blue counters. Two counters are picked without replacement. Find P(both red).

A	B	C	D
9/64	15/56	1/8	3/28

Q11 P(A) = 0.4, P(B | A) = 0.2, and P(B | A') = 0.5. Find P(A ∩ B).

A	B	C	D
0.08	0.30	0.38	0.20

Q12 Events A and B are independent with P(A) = 0.35 and P(B) = 0.60. What is P(A | B)?

A	B	C	D
0.60	0.35	0.21	0.58

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/s1-4-probability/