P23·8

Further calculus

CHAPTER ASSIGNMENT

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. Give exact values in terms of fractions, surds, π, and natural logarithms unless otherwise instructed.

Practice questions from *Cambridge International AS & A Level Mathematics: Pure Mathematics 2 & 3 Coursebook* (Cambridge University Press, Sue Pemberton and Julianne Hughes), Chapter 8 (Further calculus), Exercises 8A to 8F, End-of-chapter review exercise 8, and Cross-topic review exercise 3, for classroom use. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 8A · Derivative of $\tan^{-1} x$

8A.1 Differentiate with respect to x: (a) $\tan^{-1} 2x$ · (b) $\tan^{-1} 5x$ · (c) $\tan^{-1}(x/3)$ · (d) $\tan^{-1}(x - 1)$ · (e) $\tan^{-1} x^2$ · (f) $\tan^{-1}(2x/(x + 1))$

8A.2 Differentiate with respect to x: (a) $x \tan^{-1} x$ · (b) $(\tan^{-1} 2x)/x$ · (c) $e^x \tan^{-1} x$

8A.3 Find the equation of the tangent to the curve $y = \tan^{-1}(x/2)$ at the point where $x = 2$.

8A.4 Show that the tangent to the curve $y = \tan^{-1} x$ at the point where $x = -1$ is perpendicular to the normal to the curve at the point where $x = 1$. Find the x-coordinate of the point where this tangent and normal intersect.

Exercise 8B · Integration of $1/(x^2 + a^2)$

8B.1 Find the following integrals: (a) $\int 1/(x^2 + 9)\,dx$ · (b) $\int 1/(16 + x^2)\,dx$ · (c) $\int 1/(4x^2 + 1)\,dx$ · (d) $\int 1/(9x^2 + 16)\,dx$ · (e) $\int 1/(4x^2 + 3)\,dx$ · (f) $\int 1/(2 + 3x^2)\,dx$

8B.2 Find the exact value of each of these integrals: (a) $\int_0^3 1/(x^2 + 9)\,dx$ · (b) $\int_0^{1/2} 2/(4x^2 + 1)\,dx$ · (c) $\int_{-\sqrt{2}}^{\sqrt{2}} 1/(3x^2 + 2)\,dx$

8B.3 The curve $y = 2/\sqrt{(x^2 + 1)}$ is rotated completely about the x-axis between $x = -1$ and $x = 1$. Find the exact value of the volume of the solid formed.

Exercise 8C · Integration of k f'(x)/f(x)

8C.1 Find the following integrals: (a) $\int (6x^2)/(x^3 - 1)\,dx$ · (b) $\int (\cos x)/(1 + \sin x)\,dx$ · (c) $\int (4x - 10)/(x^2 - 5x + 1)\,dx$ · (d) $\int \cot x\,dx$ · (e) $\int x/(2 - x^2)\,dx$ · (f) $\int (\sec^2 x)/(1 + \tan x)\,dx$

8C.2 Find the exact value of each of these integrals: (a) $\int_{(\pi/6)}^{\pi/4} \tan x \, dx$ · (b) $\int_0^2 (3x^2)/(x^3 + 2) \, dx$ · (c) $\int_0^{\ln 2} e^{2x}/(1 + e^{2x}) \, dx$ · (d) $\int_1^2 (x + 1)/(x^2 + 2x - 1) \, dx$ · (e) $\int_{(\pi/6)}^{\pi/4} \cot x \, dx$ · (f) $\int_0^1 (2x + 3)/((x + 1)(x + 2)) \, dx$

8C.3 Show that $\int (e^x + e^{-x})/(e^x - e^{-x}) \, dx = \ln(1 - e^{2x}) - x + c$.

8C.4 Show that $\int_{(\pi/3)}^{\pi/4} (\sin x \cos x)/(1 - 2\cos 2x)\, dx = -1/8 \ln 2$.

8C.5 The curve $y = (4x)/(x^2 + 1)$ is defined for $x \geq 0$. Find the exact value of p for which the area enclosed by the curve, the x-axis and the line $x = p$ is equal to 4.

Exercise 8D · Integration by substitution

8D.1 Given that $I = \int x/\sqrt{(x^2 - 3)}\, dx$, use the substitution $u = x^2 - 3$ to show that $I = 1/2 \int 1/\sqrt{u}\, du$. Hence find I.

8D.2 Find these integrals, using the given substitution: (a) $\int x/(x + 2)^4\, dx$, $u = x + 2$ · (b) $\int x\sqrt{(1 - 2x^2)}\, dx$, $u = 1 - 2x^2$ · (c) $\int \cos x \sin^5 x\, dx$, $u = \sin x$ · (d) $\int e^x \sqrt{(e^x + 2)}\, dx$, $u = e^x + 2$ · (e) $\int 5x/(5x + 1)\, dx$, $u = 5x + 1$ · (f) $\int x\sqrt{(3x - 1)}\, dx$, $u = 3x - 1$

8D.3 Given that $I = \int_0^1 x^2/\sqrt{(1 - x^2)}\, dx$, use the substitution $x = \sin\theta$ to show that $I = \int_0^{\pi/2} \sin^2\theta\, d\theta$. Hence find the exact value of I.

8D.4 Find the exact value of each of these integrals: (a) $\int_0^{\pi/2} \cos^3 x\, dx$, $u = \sin x$ · (b) $\int_0^4 \sqrt{x} / (3 - \sqrt{x})\, dx$, $u = 3 - \sqrt{x}$ · (c) $\int_1^2 3 / (x(2 - \sqrt{x}))\, dx$, $u = \sqrt{x}$ · (d) $\int_0^{\sqrt{\pi}} 2x \cos(x^2/4)\, dx$, $u = \cos(x^2/4)$ · (e) $\int_{(\pi/6)}^{\pi/3} \sec^2 x \tan^3 x\, dx$, $u = \tan x$ · (f) $\int_{-2}^{2} 5/(x^2 + 4)^2\, dx$, $x = 2\tan\theta$ · (g) $\int_0^1 x^2/\sqrt{(4 - x^2)}\, dx$, $x = 2\cos\theta$ · (h) $\int_0^2 \sqrt{(x(2 - x))}\, dx$, $x = 2\sin^2\theta$

8D.5 Use the substitution $u = x - 2$ to find the exact value of $\int_1^2 4 / (1 + (x - 2)^2)\, dx$.

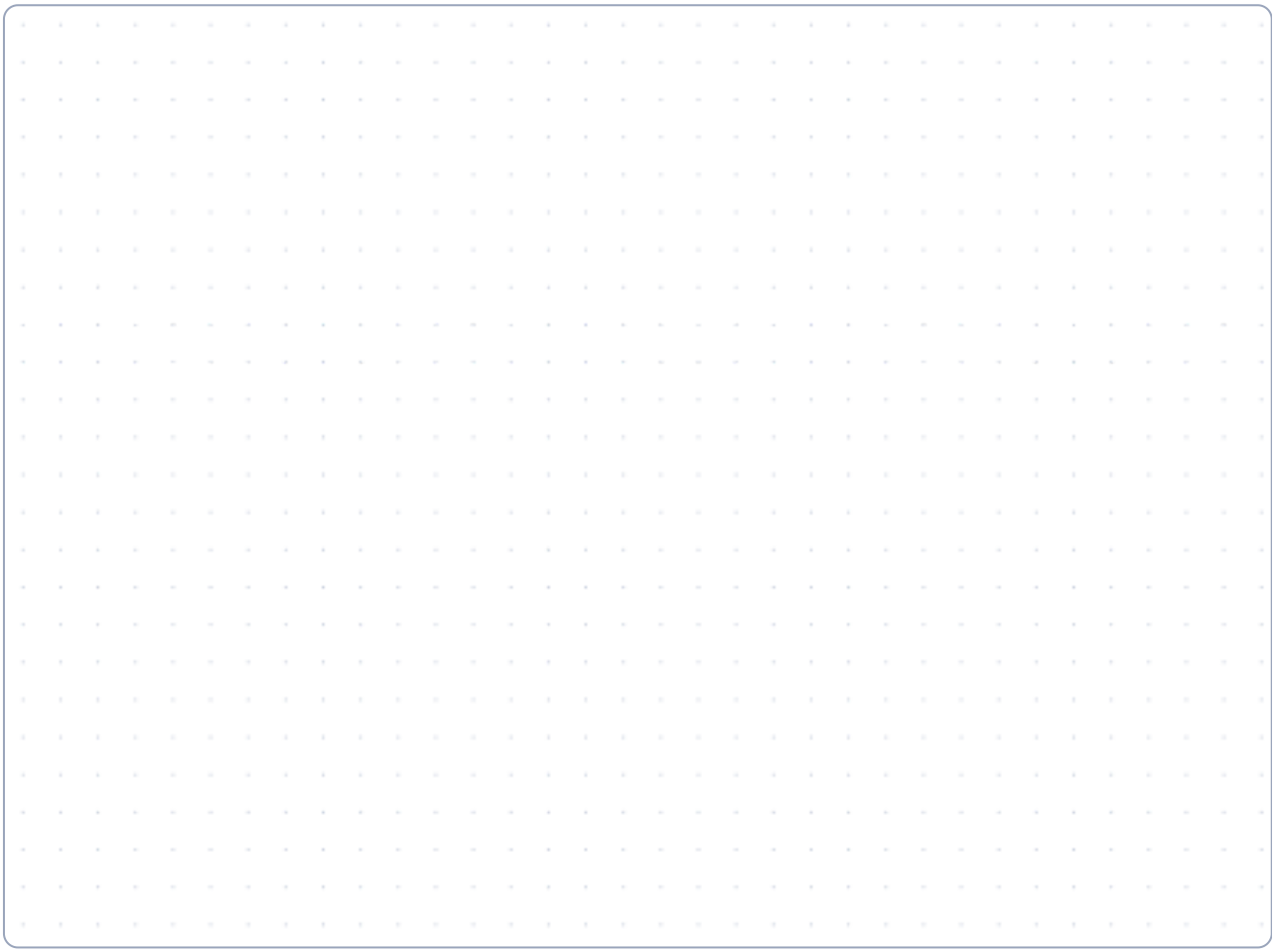
8D.6 The curve $y = 2\sin^2 x \cos^3 x$ is defined for $0 \leq x \leq \pi/2$. Using the substitution $u = \sin x$, find the exact area of the region enclosed by the curve and the x-axis.

8D.7 The curve $y = (\ln x)/\sqrt{x}$ is defined for $x \geq 1$. Using the substitution $u = \ln x$, find the exact value of the volume of the solid formed when the region bounded by the curve, the x-axis, and the line $x = e$ is rotated completely about the x-axis.

8D.8 The curve $y = 3\,e^{\cos x}\sin x$ has rotational symmetry of order 2 about the origin. Using the substitution $u = \cos x$, find the exact value of the total area bounded by the curve and the x-axis between $x = -\pi$ and $x = \pi$.

Exercise 8E · The use of partial fractions in integration

8E.1 Find each of these integrals: (a) $\int 1/(x(2 - x))\,dx$ · (b) $\int (2x - 5)/((x + 2)(x - 1))\,dx$ · (c) $\int 2(4x - 9)/((2x + 1)(x - 5))\,dx$ · (d) $\int (x^2 + 2x - 5)/((x - 3)(x^2 + 1))\,dx$ · (e) $\int (6x^2 - 19x + 20)/((x + 1)(x - 2)^2)\,dx$ · (f) $\int (2x^2 + 4x - 21)/((x - 1)(x + 4))\,dx$



8E.2 Evaluate each of these integrals: (a) $\int_1^2 (5x + 13)/((x + 2)(x + 3))\,dx$ · (b) $\int_0^2 (4x + 5)/((2x + 1)(x + 2))\,dx$ · (c) $\int_1^2 (4 - 3x)/((2x - 1)(x + 2))\,dx$ · (d) $\int_{-1}^1 (3x^2 + x)/((x - 2)(x^2 + 1))\,dx$ · (e) $\int_0^1 (x^2 + x + 2)/((x + 1)(1 + x^2))\,dx$ · (f) $\int_{-2}^0 8/((2 - x)(4 + x^2))\,dx$ · (g) $\int_2^3 (1 - x - 2x^2)/(x^2(1 - x))\,dx$ · (h) $\int_0^1 (1 - 2x)/((2x + 1)(x + 1)^2)\,dx$ · (i) $\int_1^2 (8x^2 - 3x - 2)/(4x^3 - 3x + 1)\,dx$

8E.3 By first dividing the numerator by the denominator, evaluate each of these integrals: (a) $\int_2^3 (x^2 + 2x + 3)/((x - 1)(x + 2))\,dx$ · (b) $\int_1^3 (2x^2 + 5x + 1)/((x + 1)(x + 2))\,dx$ · (c) $\int_1^2 (4x^2 + 2x - 5)/((x + 1)(2x - 1))\,dx$

8E.4 Show that $\int_{(2\sqrt{2})}^{3\sqrt{2}} (x + 3\sqrt{2})/(x^2 - 2)\,dx = \ln 3$.

8E.5 Show that $\int_0^{\pi/6} (\cos x)/(9 - \sin^2 x)\,dx = 1/6 \ln 2$.

8E.6 Show that $\int_0^{\ln 2} e^{2x} / ((1 + e^x)(2e^x + 1))\,dx = 1/2 \ln(27/20)$.

Exercise 8F · Integration by parts

8F.1 Find each of these integrals: (a) $\int 3x e^x\,dx$ · (b) $\int x \cos x\,dx$ · (c) $\int x \ln 2x\,dx$ · (d) $\int x \sin 2x\,dx$ · (e) $\int x^3 \ln x\,dx$ · (f) $\int (\ln x)/\sqrt{x}\,dx$

8F.2 Evaluate each of these integrals: (a) $\int_0^{\pi/6} x \cos 3x \, dx$ · (b) $\int_1^2 x \ln 2x \, dx$ · (c) $\int_0^1 x e^{2x} \, dx$ · (d) $\int_1^3 \ln x \, dx$ · (e) $\int_0^1 (2 - x)e^{-3x} \, dx$ · (f) $\int_{-1/5}^0 (1 - 3x)e^{-5x} \, dx$

8F.3 Use integration by parts twice to find the exact value of each of these integrals: (a) $\int_1^2 (\ln x)^2\,dx$ · (b) $\int_0^{\pi/4} x^2 \cos 2x\,dx$ · (c) $\int_0^{\pi} x^2 \sin x\,dx$ · (d) $\int_0^{e} x(\ln x)^2\,dx$ · (e) $\int_{-\infty}^{0} x^2 e^x\,dx$ · (f) $\int_0^{\pi} e^x \sin x\,dx$

8F.4 Find the exact value of the area for each of these shaded regions: (a) $y = x^2 \ln x$ between $x = 1$ and $x = e$ · (b) $y = x \sin x$ between $x = 0$ and $x = \pi$ · (c) $y = (\ln x)/x$ between $x = 1$ and $x = e$ · (d) $y = x^2 e^{-x}$ between $x = 0$ and $x = 4$

8F.5 The curve $y = e^{-x}\sqrt{(x + 2)}$ is rotated completely about the x-axis for $-2 \leq x \leq 0$. Find the exact value of the volume of the solid formed.

End-of-chapter review exercise 8

R8.1 Find the exact value of $\int_0^{1/2} x\, e^{-2x}\, dx$. [Cambridge International A Level Mathematics 9709 Paper 31 Q2 June 2016]

R8.2 Find the exact value of $\int_1^4 (\ln x)/\sqrt{x}\, dx$. [Cambridge International A Level Mathematics 9709 Paper 31 Q3 November 2013]

R8.3 Use the substitution $u = 1 + 3\tan x$ to find the exact value of $\int_0^{\pi/4} \sqrt{(1 + 3\tan x)} / \cos^2 x\, dx$. [Cambridge International A Level Mathematics 9709 Paper 31 Q2 June 2014]

R8.4 (i) Prove that $\cot\theta + \tan\theta \equiv 2\operatorname{cosec} 2\theta$. (ii) Hence show that $\int_{(\pi/6)}^{\pi/3} \operatorname{cosec} 2\theta \, d\theta = \frac{1}{2} \ln 3$. [Cambridge International A Level Mathematics 9709 Paper 31 Q5 November 2013]

R8.5 Let $I = \int_0^1 x^2/\sqrt{(4 - x^2)} \, dx$. (i) Using the substitution $x = 2\sin\theta$, show that $I = \int_0^{\pi/6} 4\sin^2\theta \, d\theta$. (ii) Hence find the exact value of I. [Cambridge International A Level Mathematics 9709 Paper 31 Q5 November 2010]

R8.6 The integral I is defined by $I = \int_0^2 4t^3 \ln(t^2 + 1) \, dt$. (i) Use the substitution $x = t^2 + 1$ to show that $I = \int_1^5 (2x - 2) \ln x \, dx$. (ii) Hence find the exact value of I. [Cambridge International A Level Mathematics 9709 Paper 31 Q7 June 2011]

R8.7 (i) Express $2/((x + 1)(x + 3))$ in partial fractions. (ii) Using your answer to part i, show that $(2/((x + 1)(x + 3)))^2 = 1/(x + 1)^2 - 1/(x + 1) + 1/(x + 3) + 1/(x + 3)^2$. (iii) Hence show that $\int_0^1 4/((x + 1)^2(x + 3)^2)\, dx = 7/12 - \ln(3/2)$. [Cambridge International A Level Mathematics 9709 Paper 31 Q8 June 2010]

R8.8 (a) Find $\int (4 + \tan^2 2x)\, dx$. (b) Find the exact value of $\int_{(\pi/4)}^{\pi/2} (\sin(x + \pi/6))/(\sin x)\, dx$. [Cambridge International A Level Mathematics 9709 Paper 31 Q5 June 2015]

R8.9 (i) Prove the identity $\tan 2\theta - \tan \theta \equiv \tan \theta \sec 2\theta$. (ii) Hence show that $\int_0^{\pi/6} \tan \theta \sec 2\theta\, d\theta = 1/2 \ln(3/2)$. [Cambridge International A Level Mathematics 9709 Paper 31 Q5 November 2016]

R8.10 The curve $y = (2x - x^2)e^{x/2}$ has a maximum point M. (i) Find the exact x-coordinate of M. (ii) Find the exact value of the area of the shaded region bounded by the curve and the positive x-axis. [Cambridge International A Level Mathematics 9709 Paper 31 Q7 November 2016]

R8.11 By first expressing $(4x^2 + 5x + 3)/(2x^2 + 5x + 2)$ in partial fractions, show that $\int_0^4 (4x^2 + 5x + 3)/(2x^2 + 5x + 2)\, dx = 8 - \ln 9$. [Cambridge International A Level Mathematics 9709 Paper 31 Q9 June 2012]

R8.12 The curve $y = x^3 \ln x$ has a minimum point M. (i) Find the exact coordinates of M. (ii) Find the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 2$. [Cambridge International A Level Mathematics 9709 Paper 31 Q9 November 2010]

R8.13 The curve $y = x^2/(1 + x^3)$ for $x \geq 0$ has a maximum point M. The region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = p$. (i) Find the exact value of the x-coordinate of M. (ii) Calculate the value of p for which the area of R is equal to 1. Give your answer correct to 3 significant figures. [Cambridge International A Level Mathematics 9709 Paper 31 Q10 November 2015]

Cross-topic review exercise 3

CR3.1 Expand $(1 - 3x)^{-5}$ in ascending powers of x, up to and including the term in x^3.

CR3.2 Evaluate $\int_0^1 1/(3x^2 + 1)\, dx$.

CR3.3 Expand $32/(x + 2)^5$ in ascending powers of x, up to and including the term in x^3.

CR3.4 Expand $1/\sqrt{(4 - 2x)}$ in ascending powers of x, up to and including the term in x^2.

CR3.5 Evaluate $\int_0^{\pi/2} x^2 \sin x \, dx$.

CR3.6 Expand $\sqrt{((1 + 2x)/(1 - x))}$ in ascending powers of x, up to and including the term in x^3.

CR3.7 Show that $\int_0^1 (x + 2)e^{-2x} \, dx = 5/4 - 7/(4e^2)$.

CR3.8 (i) Expand $1/\sqrt{(1 - 4x)}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. (ii) Hence find the coefficient of x^2 in the expansion of $(1 + 2x)/\sqrt{(4 - 16x)}$. [Cambridge International A Level Mathematics 9709 Paper 31 Q2 June 2012]

CR3.9 When $(1 + ax)^{-2}$, where a is a positive constant, is expanded in ascending powers of x, the coefficients of x and x^3 are equal. (i) Find the exact value of a. (ii) When a has this value, obtain the expansion up to and including the term in x^2, simplifying the coefficients. [Cambridge International A Level Mathematics 9709 Paper 31 Q4 November 2012]

CR3.10 The curve $y = 8 \sin(x/2) - \tan(x/2)$ is defined for $0 \leq x < \pi$. The x-coordinate of the maximum point is α and the shaded region is enclosed by the curve and the lines $x = \alpha$ and $y = 0$. (i) Show that $\alpha = 2/3\ \pi$. (ii) Find the exact value of the area of the shaded region. [Cambridge International A Level Mathematics 9709 Paper 31 Q5 June 2012]

CR3.11 Show that $\int_0^5 (5 - 3x)/((x + 1)(3x + 1))\ dx = 4 \ln(4/3)$.

CR3.12 Let $I = \int_0^{1/2} 4x^2/\sqrt{(1 - x^2)}\, dx$. (a) Using the substitution $x = \sin\theta$, show that $I = \int_0^{\pi/6} 4\sin^2\theta\, d\theta$. (b) Hence show that $I = \pi/3 - \sqrt{3}/2$.

CR3.13 It is given that $\int_1^a x \ln x\, dx = 30$, where $a > 1$. (a) Show that $a = \sqrt{(119 / (2 \ln a - 1))}$. (b) Use the iterative formula $a_{(n+1)} = \sqrt{(119 / (2 \ln a_n - 1))}$ to determine the value of a correct to 2 decimal places. Give the result of each iteration correct to 4 decimal places.

CR3.14 The curve $y = x^2 e^{2-x}$ has a maximum point M. (i) Show that the x-coordinate of M is 2. (ii) Find the exact value of $\int_0^2 x^2 e^{2-x}\, dx$. [Cambridge International A Level Mathematics 9709 Paper 31 Q9 June 2015]

CR3.15 Let $f(x) = (12 + 8x - x^2)/((2 - x)(4 + x^2))$. (i) Express $f(x)$ in the form $A/(2 - x) + (Bx + C)/(4 + x^2)$. (ii) Show that $\int_0^1 f(x)\,dx = \ln(25/2)$. [Cambridge International A Level Mathematics 9709 Paper 31 Q8 November 2011]

CR3.16 It is given that $\int_1^a \ln(2x)\,dx = 1$, where $a > 1$. (i) Show that $a = 1/2 \exp(1 + (\ln 2)/a)$, where $\exp(x)$ denotes e^x. (ii) Use the iterative formula $a_{n+1} = 1/2 \exp(1 + (\ln 2)/a_n)$ to determine the value of a correct to 2 decimal places. Give the result of each iteration correct to 4 decimal places. [Cambridge International A Level Mathematics 9709 Paper 31 Q6 November 2014]

CR3.17 (i) Express $(4 + 12x + x^2)/((3 - x)(1 + 2x)^2)$ in partial fractions. (ii) Hence obtain the expansion of $(4 + 12x + x^2)/((3 - x)(1 + 2x)^2)$ in ascending powers of x, up to and including the term in x^2. [Cambridge International A Level Mathematics 9709 Paper 31 Q9 June 2014]

CR3.18 Let $f(x) = (x^2 - 8x + 9)/((1 - x)(2 - x)^2)$. (i) Express $f(x)$ in partial fractions. (ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [Cambridge International A Level Mathematics 9709 Paper 31 Q9 November 2014]

CR3.19 Let $f(x) = (5x - 2)/((x - 1)(2x^2 - 1))$. (a) Express $f(x)$ in partial fractions. (b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3.

CR3.20 (a) Given that $(x^3 - 2)/(x^2(2x - 1)) \equiv A + B/x + C/x^2 + D/(2x - 1)$, find the values of the constants A, B, C and D. (b) Hence show that $\int_1^2 (x^3 - 2)/(x^2(2x - 1))\, dx = 3/2 - 2 \ln(9/4) + 1/4 \ln 3$.

CR3.21 The curve $y = x^2 \ln x$ has a minimum point M. (i) Find the exact values of the coordinates of M. (ii) Find the exact value of the area of the shaded region bounded by the curve, the x-axis and the line $x = e$. [Cambridge International A Level Mathematics 9709 Paper 31 Q9 November 2011]

CR3.22 (i) Show that $\int_2^4 4x \ln x \, dx = 56 \ln 2 - 12$. (ii) Use the substitution $u = \sin 4x$ to find the exact value of $\int_0^{\pi/24} \cos^3 4x \, dx$. [Cambridge International A Level Mathematics 9709 Paper 31 Q8 June 2013]

CR3.23 (i) Express $4 \cos \theta + 3 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 1/2 \, \pi$. Give the value of α correct to 4 decimal places. (ii) Hence (a) solve the equation $4 \cos \theta + 3 \sin \theta = 2$ for $0 < \theta < 2\pi$, (b) find $\int 50 / (4 \cos \theta + 3 \sin \theta)^2 \, d\theta$. [Cambridge International A Level Mathematics 9709 Paper 31 Q9 June 2013]

Check yourself

C.1 Differentiate $y = \tan^{-1}(2x^3)$ with respect to x.

C.2 Find the exact value of the definite integral $\int_0^{1/\sqrt{3}} 1/(1 + 9x^2)\, dx$.

C.3 Use integration by parts to evaluate $\int_1^e x^2 \ln x\, dx$.

C.4 Express $(3x + 1)/((x - 1)(x + 1))$ in partial fractions and hence evaluate $\int_2^3 (3x + 1)/((x - 1)(x + 1))\, dx$.

C.5 Use the substitution $u = 1 + x^2$ to find the exact value of $\int_0^1 x^3 / \sqrt{(1 + x^2)}\, dx$.

Chapter quiz

Q1 What is the derivative of $y = \tan^{-1}(3x)$ with respect to x?

A	B	C	D
$1/(1 + 9x^2)$	$3/(1 + 3x^2)$	$3/(1 + 9x^2)$	$9/(1 + 9x^2)$

Q2 What is the derivative of $y = \tan^{-1}(x/a)$ where a is a non-zero constant?

A	B	C	D
$a/(x^2 + a^2)$	$1/(x^2 + a^2)$	$1/(a(x^2 + 1))$	$a^2/(x^2 + a^2)$

Q3 Find the indefinite integral $\int 1/(x^2 + 16)\, dx$.

A	B	C	D
$(1/16) \tan^{-1}(x/4) + c$	$(1/4) \tan^{-1}(x/4) + c$	$(1/4) \tan^{-1}(4x) + c$	$4 \tan^{-1}(x/4) + c$

Q4 Evaluate the definite integral $\int_0^3 1/(x^2 + 9)\, dx$.

A	B	C	D
$\pi/4$	$\pi/3$	$1/12$	$\pi/12$

Q5 Evaluate the indefinite integral $\int (6x^2)/(x^3 - 5)\,dx$.

A	B	C	D
$2 \ln \lvert x^3 - 5 \rvert + c$	$6 \ln \lvert x^3 - 5 \rvert + c$	$3 \ln \lvert x^3 - 5 \rvert + c$	$(2/3) \ln \lvert x^3 - 5 \rvert + c$

Q6 Which of the following is equal to $\int \cot(2x)\,dx$?

A	B	C	D
$-(1/2) \ln \lvert \sin 2x \rvert + c$	$(1/2) \ln \lvert \cos 2x \rvert + c$	$(1/2) \ln \lvert \sin 2x \rvert + c$	$2 \ln \lvert \sin 2x \rvert + c$

Q7 Find the integral $\int \cos x \sin^4 x\,dx$.

A	B	C	D
$(1/5) \cos^5 x + c$	$(1/5) \sin^5 x + c$	$-(1/5) \sin^5 x + c$	$(1/4) \sin^5 x + c$

Q8 Using the substitution $u = 2x + 1$, what does the integral $\int 4x\sqrt{(2x + 1)}\,dx$ become?

A	B	C	D
$\int (u^{3/2} - u^{1/2})\,du$	$\int (2u^{3/2} - 2u^{1/2})\,du$	$\int (u^{3/2} + u^{1/2})\,du$	$\int (1/2)(u^{3/2} - u^{1/2})\,du$

Q9 Using substitution $x = 3 \sin \theta$, evaluate $\int_0^3 \sqrt{(9 - x^2)}\,dx$.

A	B	C	D
$9\pi/2$	$3\pi/4$	9π	$9\pi/4$

Q10 Decomposing $2/(x^2 - 1)$ into partial fractions gives which expression to integrate?

A	B	C	D
$1/(x - 1) + 1/(x + 1)$	$1/(x - 1) - 1/(x + 1)$	$2/(x - 1) - 2/(x + 1)$	$1/(x + 1) - 1/(x - 1)$

Q11 Applying integration by parts $\int u\,(dv/dx)\,dx$ to $\int x\,e^{2x}\,dx$, what is the best choice for u?

A	B	C	D
$u = x$	$u = e^{2x}$	$u = x^2$	$u = 2x$

Q12 What is the indefinite integral $\int \ln x \, dx$?

A	B	C	D
$x \ln x + x + c$	$(1/x) + c$	$x \ln x - x + c$	$(1/2)(\ln x)^2 + c$

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/p23-8-further-calculus/