P23·5

Integration

CHAPTER ASSIGNMENT · 33 MARKS

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. Give exact answers (fractions, surds, π, e, logarithms) unless a question asks for decimal places or significant figures. ∫(a to b) f(x) dx means the definite integral of f(x) from x = a to x = b, and ∫ f(x) dx with no limits is an indefinite integral. All angles in calculus questions are in radians.

Practice questions from *Cambridge International AS & A Level Mathematics: Pure Mathematics 2 & 3 Coursebook* (Cambridge University Press, Sue Pemberton and Julianne Hughes), Chapter 5 (Integration), including Prerequisite knowledge, Explore 5.1 and 5.2, Exercises 5A to 5E, and End-of-chapter review exercise 5, for classroom use. Where a question needs a diagram, the diagram is described in words. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Prerequisite knowledge & Explore tasks

5.P1 Find dy/dx when: (a) $y = 3\sin 2x - 5\cos x$ · (b) $y = e^{5x - 2}$ · (c) $y = \ln(2x + 1)$.

5.P2 Prove that $\cos 4x + 4\cos 2x \equiv 8\cos^4 x - 3$.

5.E1 Explore 5.1: Fiona is asked to find $\int x^{-1}\,dx$. She tries to use the formula $\int x^n\,dx = (1/(n + 1))x^{n + 1} + c$ to obtain her answer. Fiona is also asked to find $\int (2x + 3)^{-1}\,dx$. She tries to use the formula $\int (ax + b)^n\,dx = (1/(a(n + 1)))(ax + b)^{n + 1} + c$ to obtain her answer. Explain why Fiona's methods do not work.

5.E2 Explore 5.2: Rafiu and Fausat are asked to find $\int 1/(2(3x + 1))\,dx$. Rafiu writes: $\int 1/(2(3x + 1))\,dx = \int 1/(6x + 2)\,dx = (1/6)\ln(6x + 2) + c$. Fausat writes: $\int 1/(2(3x + 1))\,dx = (1/2)\int 1/(3x + 1)\,dx = (1/2)(1/3)\ln(3x + 1) + c = (1/6)\ln(3x + 1) + c$. Decide who is correct and explain the reasons for your decision.

Exercise 5A · Integration of exponential functions

5A.1 Find: (a) $\int e^{2x}\,dx$ · (b) $\int e^{-4x}\,dx$ · (c) $\int 6e^{3x}\,dx$ · (d) $\int 4e^{(1/2)x}\,dx$ · (e) $\int 2e^{-x}\,dx$ · (f) $\int e^{2x + 4}\,dx$ · (g) $\int e^{3x - 1}\,dx$ · (h) $\int 6e^{2 - 3x}\,dx$ · (i) $\int 2e^{8x - 3}\,dx$

5A.2 Find: (a) $\int e^{-x}(1 + e^{x})\,dx$ · (b) $\int 5e^{x}(2 + e^{3x})\,dx$ · (c) $\int (e^{2x} - 1)^{2}\,dx$ · (d) $\int (4 + e^{2x})/e^{2x}\,dx$ · (e) $\int (8 - e^{x})/(2e^{2x})\,dx$ · (f) $\int (e^{x} - 2)^{2}/e^{2x}\,dx$

5A.3 Evaluate: (a) $\int_0^2 e^{3x}\,dx$ · (b) $\int_0^{1/2} e^{4x}\,dx$ · (c) $\int_0^{\ln 2} 5e^{-2x}\,dx$ · (d) $\int_0^3 e^{1+2x}\,dx$ · (e) $\int_0^1 8/e^{2x-1}\,dx$ · (f) $\int_0^1 (e^x+1)^2\,dx$ · (g) $\int_0^1 (e^x+e^{2x})^2\,dx$ · (h) $\int_0^1 (2e^x-5/e^x)^2\,dx$ · (i) $\int_0^2 (e^x+1)^2/e^{2x}\,dx$

5A.4 A curve is such that $dy/dx = 6e^{2x} + 2e^{-x}$. Given that the curve passes through the point (0, 2), find the equation of the curve.

5A.5 A curve is such that $d^2y/dx^2 = 20e^{-2x}$. Given that $dy/dx = -8$ when $x = 0$ and that the curve passes through the point $(1, 5/e^2)$, find the equation of the curve.

5A.6 Find the exact area of the region bounded by the curve $y = 1 + e^{2x - 5}$, the x-axis and the lines $x = 1$ and $x = 3$.

5A.7 The diagram shows the curve $y = 2e^{(1/2)x} - 2x + 3$. The shaded region is bounded by the curve, the y-axis, the x-axis and the line $x = 3$. Find the exact area of the shaded region.

5A.8 (a) Show that $d/dx\ (xe^x - e^x) = xe^x$. (b) The diagram shows the curve $y = xe^x$, with the region under the curve between $x = 0$ and $x = 3$ shaded. Use your result from part a to evaluate the area of the shaded region.

5A.9 (a) Find $\int_0^a (4e^{-2x} + 5e^{-x})\,dx$, where a is a positive constant. (b) Hence find the value of $\int_0^\infty (4e^{-2x} + 5e^{-x})\,dx$.

5A.10 The diagram shows the curve $y = 2e^x + 8e^{-x} - 7$ and its minimum point M. The shaded region is bounded by the curve, the y-axis, the x-axis and the vertical line through M. Find the area of the shaded region.

5A.11 The diagram shows the graph of $y = e^x$. The points (ln 2, 2) and (ln 3, 3) lie on the curve. (a) Find the value of $\int_{\ln 2}^{\ln 3} e^x \, dx$. (b) Hence show that $\int_2^3 \ln y \, dy = \ln(27/(4e))$.

Exercise 5B · Integration of 1/(ax + b)

5B.1 Find: (a) $\int 6/x \, dx$ · (b) $\int 1/(2x) \, dx$ · (c) $\int 1/(3x + 1) \, dx$ · (d) $\int 6/(2x - 5) \, dx$ · (e) $\int 5/(2 - 3x) \, dx$ · (f) $\int 3/(2(5x - 1)) \, dx$

5B.2 Evaluate: (a) $\int_0^5 1/(x + 2) \, dx$ · (b) $\int_1^4 1/(2x + 1) \, dx$ · (c) $\int_{-1}^4 3/(2x + 5) \, dx$ · (d) $\int_2^4 2/(4x + 1) \, dx$ · (e) $\int_0^2 3/(2x - 7) \, dx$ · (f) $\int_2^3 4/(3 - 2x) \, dx$

5B.3 Evaluate: (a) $\int_4^{10} \left(2 + \frac{5}{3x - 2}\right) dx$ · (b) $\int_1^3 \left(\frac{3}{x} - \frac{4}{2x - 1}\right) dx$ · (c) $\int_{-1}^3 \left(2x - 1 + \frac{4}{2x + 3}\right) dx$

5B.4 (a) Given that $\frac{4x}{2x - 1} \equiv 2 + \frac{A}{2x - 1}$, find the value of the constant A. (b) Hence show that $\int_1^5 \frac{4x}{2x - 1} dx = 8 + \ln 9$.

5B.5 (a) Find the quotient and remainder when $6x^2 + 5x$ is divided by $2x - 5$. (b) Hence show that $\int_0^1 \frac{6x^2 + 5x}{2x - 5} dx = \frac{23}{2} - 25 \ln\left(\frac{5}{3}\right)$.

5B.6 A curve is such that $dy/dx = 2x + 3/(x + e)$. Given that the curve passes through the point (e, e^2), find the equation of the curve.

5B.7 The diagram shows part of the curve $y = 2/(x + 3)$, with the region under the curve between $x = 1$ and $x = k$ shaded. Given that the shaded region has area 4, find the value of k.

5B.8 The points P(1, −2) and Q(2, k) lie on the curve for which $dy/dx = 3 - 2/x$. The tangents to the curve at the points P and Q intersect at the point R. Find the coordinates of R.

Exercise 5C · Integration of sin(ax + b), cos(ax + b) and sec²(ax + b)

5C.1 Find: (a) $\int \sin 3x\,dx$ · (b) $\int \cos 4x\,dx$ · (c) $\int \sin(x/2)\,dx$ · (d) $\int 3\sin 2x\,dx$ · (e) $\int 5\cos 3x\,dx$ · (f) $\int \sec^2 2x\,dx$ · (g) $\int 2\cos(1-5x)\,dx$ · (h) $\int 3\sin(2x+1)\,dx$ · (i) $\int 2\sec^2(5x-2)\,dx$

5C.2 Evaluate: (a) $\int_0^{(1/6)\pi} \cos 4x\,dx$ · (b) $\int_0^{(2/3)\pi} \sin((1/2)x)\,dx$ · (c) $\int_0^{(1/6)\pi} \sec^2 2x\,dx$ · (d) $\int_0^{(1/2)\pi} (\sin 2x + \cos x)\,dx$ · (e) $\int_0^{(1/6)\pi} (\cos 2x + \sin x)\,dx$ · (f) $\int_{-(1/4)\pi}^{(1/4)\pi} (5 - 2\sec^2 x)\,dx$

5C.3 (a) Find d/dx (x sin x + cos x). (b) Hence find the exact value of $\int_0^{\frac{1}{3}\pi} x \cos x \, dx$.

5C.4 A curve is such that $\frac{dy}{dx} = 1 - 3\sin 2x$. Given that the curve passes through the point $(\frac{\pi}{4}, 0)$, find the equation of the curve.

5C.5 A curve is such that $\frac{d^2y}{dx^2} = -12\sin 2x - 2\cos x$. Given that $\frac{dy}{dx} = 4$ when $x = 0$ and that the curve passes through the point $(\frac{\pi}{2}, -3)$, find the equation of the curve.

5C.6 The point $(\frac{\pi}{2}, 5)$ lies on the curve for which $\frac{dy}{dx} = 4\sin(2x - \frac{\pi}{2})$. (a) Find the equation of the curve. (b) Find the equation of the normal to the curve at the point where $x = \frac{\pi}{3}$.

5C.7 The diagram shows part of the curve $y = 1 + \sqrt{3}\sin 2x + \cos 2x$. The shaded region is bounded by the curve, the y-axis and the positive x-axis up to the first positive x-intercept. Find the exact value of the area of the shaded region.

5C.8 The diagram shows part of the curve $y = 3\sin 2x + 6\sin x$ (for $0 \leq x \leq \pi$) and its maximum point M. The shaded region is bounded by the curve, the x-axis and the vertical line through M (from x = 0 to M). Find the exact area of the shaded region.

5C.9 The diagram shows part of the graph of $y = \sin x$. The points $(\pi/6, 1/2)$ and $(\pi/3, \sqrt{3}/2)$ lie on the curve. (a) Find the exact value of $\int_{\pi/6}^{\pi/3} \sin x \, dx$. (b) Hence show that $\int_{1/2}^{\sqrt{3}/2} (\sin^{-1} y) \, dy = (\pi/12)(2\sqrt{3} - 1) - (\sqrt{3} - 1)/2$.

5C.10 The diagram shows part of the graph of $y = \cos x$. The points $(\pi/4, \sqrt{2}/2)$ and $(\pi/3, 1/2)$ lie on the curve. (a) Find the exact value of $\int_{\pi/4}^{\pi/3} \cos x \, dx$. (b) Hence show that $\int_{1/2}^{\sqrt{2}/2} (\cos^{-1} y) \, dy = (\pi/24)(3\sqrt{2} - 4) + (\sqrt{3} - \sqrt{2})/2$.

Exercise 5D · Further integration of trigonometric functions

5D.1 Find: (a) $\int 3\cos^2 x \, dx$ · (b) $\int 4\cos^2(x/2) \, dx$ · (c) $\int \sin^2 3x \, dx$ · (d) $\int 2\tan^2 x \, dx$ · (e) $\int 6\tan^2(3x) \, dx$ · (f) $\int \cos^4 x \, dx$

5D.2 Find the value of: (a) $\int_0^{\frac{1}{3}\pi} \sin^2 x\,dx$ · (b) $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 3\tan^2 x\,dx$ · (c) $\int_0^{\frac{1}{3}\pi} \cos^2 x\,dx$ · (d) $\int_0^{\frac{1}{8}\pi} \cos^2 2x\,dx$ · (e) $\int_0^{\frac{1}{6}\pi} 4\sin^2 x\,dx$ · (f) $\int_0^{\frac{1}{6}\pi} \tan^2 2x\,dx$

5D.3 Find the value of: (a) $\int_0^{\frac{1}{6}\pi} (\cos^2 x - 1/\cos^2 x)\,dx$ · (b) $\int_0^{\frac{1}{6}\pi} (\cos^2 x + \sin 2x)\,dx$ · (c) $\int_0^{\frac{1}{4}\pi} (2\sin x + \cos x)^2\,dx$ · (d) $\int_0^{\frac{1}{3}\pi} (\cos x - 3\sin x)^2\,dx$ · (e) $\int_0^{\frac{1}{6}\pi} (1 + \cos^4 x)/\cos^2 x\,dx$ · (f) $\int_0^{\frac{1}{6}\pi} 5/(1 + \cos 4x)\,dx$

5D.4 (a) By differentiating cos x / sin x, show that if y = cot x then dy/dx = $-\text{cosec}^2$ x. (b) Hence, show that $\int((1/6)\pi$ to $(1/4)\pi)$ cosec^2 2x dx = $\sqrt{3}/6$.

5D.5 (a) Show that tan x + cot x ≡ 2/sin 2x. (b) Hence show that $\int((1/3)\pi$ to $(1/2)\pi)$ 8/(tan x + cot x) dx = 1.

5D.6 (a) Use the expansions of sin(5x + 2x) and sin(5x − 2x) to show that: sin 7x + sin 3x ≡ 2 sin 5x cos 2x. (b) Hence show that $\int((1/6)\pi$ to $(1/3)\pi)$ 2 sin 5x cos 2x dx = $(11 - 3\sqrt{3})/42$.

5D.7 (a) Show that $\sin 3x \equiv 3 \sin x - 4 \sin^3 x$. (b) Hence show that $\int_0^{\frac{1}{3}\pi} 2 \sin^3 x \, dx = \frac{5}{12}$.

5D.8 (a) Show that $\cos 3x \equiv 4 \cos^3 x - 3 \cos x$. (b) Hence show that $\int_0^{\frac{1}{6}\pi} (4 \cos^3 x + 2 \cos x) \, dx = \frac{17}{6}$.

5D.9 The diagram shows part of the curve $y = \cos x + 2 \sin x$, with the region under the curve between $x = 0$ and $x = \frac{1}{2}\pi$ shaded. Find the exact volume of the solid formed when the shaded region is rotated through 360° about the x-axis.

5D.10 (a) Show that $\sec^2 x + \sec x \tan x \equiv 1/(1 - \sin x)$. (b) Show that if $y = \sec x$ then $dy/dx = \sec x \tan x$. (c) Hence show that $\int_0^{(1/6)\pi} 4/(1 - \sin 2x)\,dx = 2(1 + \sqrt{3})$.

Exercise 5E · The trapezium rule (P2 only)

5E.1 Use the trapezium rule with 2 intervals to estimate the value of each of these definite integrals. Give your answers correct to 2 decimal places. (a) $\int_2^4 \sqrt{(x^2 - 2)}\,dx$ · (b) $\int_0^2 |e^x - 12|\,dx$ · (c) $\int_0^4 2/(3 + e^x)\,dx$ · (d) $\int_1^7 2/\ln(x + 5)\,dx$ · (e) $\int_0^{(1/3)\pi} 1/(2 + \tan x)\,dx$ · (f) $\int_2^{12} \log_{10} x\,dx$

5E.2 The diagram shows part of the curve $y = x^2e^{-x}$. Use the trapezium rule with 4 intervals to estimate the value of $\int_1^5 x^2e^{-x}\,dx$, giving your answer correct to 2 decimal places.

5E.3 (a) Use the trapezium rule with 6 intervals to estimate the value of $\int_{(1/4)\pi}^{(7/4)\pi} \operatorname{cosec}((1/2)x)\,dx$, giving your answer correct to 2 decimal places. (b) Use a sketch of the graph of $y = \operatorname{cosec}((1/2)x)$ for $0 < x < 2\pi$, to explain whether the trapezium rule gives an under-estimate or an over-estimate of the true value of $\int_{(1/4)\pi}^{(7/4)\pi} \operatorname{cosec}((1/2)x)\,dx$.

5E.4 The diagram shows the curve $y = e^x \cos x$ for $0 \le x \le (1/2)\pi$ (it rises from (0, 1) to a maximum and falls to $((1/2)\pi, 0)$, bending downwards throughout). Use the trapezium rule with 3 intervals to estimate the value of $\int_0^{(1/2)\pi} e^x \cos x\,dx$, giving your answer correct to 2 decimal places. State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of $\int_0^{(1/2)\pi} e^x \cos x\,dx$.

5E.5 The diagram shows part of the curve $y = e^x/(2x)$, with the region under the curve between $x = 1$ and $x = 3$ shaded (the curve bends upwards). Use the trapezium rule with 4 intervals to estimate the area of the shaded region, giving your answer correct to 2 decimal places. State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the shaded area.

End-of-chapter review exercise 5

R5.1 Show that $\int_{(1/6)\pi}^{(1/3)\pi} (1 + 2\tan^2 x)\, dx = 4\sqrt{3}/3 - \pi/6$. [4]

R5.2 Find the exact value of $\int_{-4}^{-2} (4/(1 - 2x) - 2x)\, dx$, giving your answer in the form $a + \ln(b/c)$, where a, b and c are integers. [4]

R5.3 The diagram shows part of the curve $y = 2 - x^2 \ln(x + 1)$ (falling from (0, 2) and bending downwards). The shaded region R is bounded by the curve and by the lines $x = 0$, $x = 1$ and $y = 0$. Use the trapezium rule with 4 intervals to estimate the area of R giving your answer correct to 2 decimal places. State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the area of R. [4]

R5.4 Find the exact value of $\int_0^{(1/6)\pi} (\cos 3x - \sin 2x)\, dx$. [5]

R5.5 Show that $\int_0^1 (3e^x - 2)^2\, dx = (9/2)e^2 - 12e + 23/2$. [5]

R5.6 (a) Find $\int_0^k (5e^{-2x} + 2e^{-3x})\, dx$, where k is a positive constant. [4] (b) Hence find the exact value of $\int_0^\infty (5e^{-2x} + 2e^{-3x})\, dx$. [1]

R5.7 Show that $\int_2^4 \frac{3}{5x+1}\,dx = \frac{3}{5}\ln\frac{21}{11}$. [5]

R5.8 (a) Find the value of the constant A such that $\frac{8x}{2x+5} \equiv 4 + \frac{A}{2x+5}$. [2] (b) Hence show that $\int_1^3 \frac{8x}{2x+5}\,dx = 8 - 10\ln\frac{11}{7}$. [5]

R5.9 (i) Show that $12\sin^2 x\cos^2 x \equiv \frac{3}{2}(1-\cos 4x)$. [3] (ii) Hence show that $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} 12\sin^2 x\cos^2 x\,dx = \frac{\pi}{8} + \frac{3\sqrt{3}}{16}$. [3] [Cambridge International AS & A Level Mathematics 9709 Paper 21 Q3 June 2013]

R5.10 (a) Find $\int 4e^{x}(3 + e^{2x})\,dx$. [3] (b) Show that $\int_{-(1/4)\pi}^{(1/4)\pi} (3 + 2\tan^2\theta)\,d\theta = (1/2)(8 + \pi)$. [4] [Cambridge International AS & A Level Mathematics 9709 Paper 21 Q6 June 2011]

R5.11 (i) Show that $(2\sin x + \cos x)^2$ can be written in the form $5/2 + 2\sin 2x - (3/2)\cos 2x$. [5] (ii) Hence find the exact value of $\int_{0}^{(1/4)\pi} (2\sin x + \cos x)^2\,dx$. [4] [Cambridge International AS & A Level Mathematics 9709 Paper 21 Q7 June 2012]

R5.12 (i) By differentiating cos x / sin x, show that if y = cot x then dy/dx = −cosec^2 x. [3] (ii) By expressing cot^2 x in terms of cosec^2 x and using the result of part i, show that ∫((1/4)π to (1/2)π) cot^2 x dx = 1 − (1/4)π. [4] (iii) Express cos 2x in terms of sin^2 x and hence show that 1/(1 − cos 2x) can be expressed as (1/2) cosec^2 x. Hence, using the result of part i, find ∫1/(1 − cos 2x) dx. [3] [Cambridge International AS & A Level Mathematics 9709 Paper 21 Q8 June 2010]

R5.13 (i) Find ∫(1 + cos^4 2x)/cos^2 2x dx. [3] (ii) Without using a calculator, find the exact value of ∫(4 to 14) (2 + 6/(3x − 2)) dx, giving your answer in the form ln(ae^b), where a and b are integers. [5] [Cambridge International AS & A Level Mathematics 9709 Paper 21 Q7 June 2016]

R5.14 The diagram shows the curve $y = x \sin x$, for $0 \leq x \leq \pi$. The point $Q((1/2)\pi, (1/2)\pi)$ lies on the curve. (i) Show that the normal to the curve at Q passes through the point $(\pi, 0)$. [5] (ii) Find $d/dx (\sin x - x \cos x)$. [2] (iii) Hence evaluate $\int_0^{(1/2)\pi} x \sin x \, dx$. [3] [Cambridge International AS & A Level Mathematics 9709 Paper 21 Q8 November 2010]

R5.15 (i) Using the expansion of $\cos(3x - x)$ and $\cos(3x + x)$, prove that $(1/2)(\cos 2x - \cos 4x) \equiv \sin 3x \sin x$. [3] (ii) Hence show that $\int_{(1/6)\pi}^{(1/3)\pi} \sin 3x \sin x \, dx = (1/8)\sqrt{3}$. [3] [Cambridge International A Level Mathematics 9709 Paper 31 Q4 June 2010]

Check yourself

C.1 Find $\int (6e^{3x} - 4e^{-2x} + 8/(4x + 1)) \, dx$.

C.2 Evaluate $\int_0^2 (10x + 1)/(2x + 1)\, dx$, giving your answer in the form $a - b \ln c$.

C.3 Find the exact value of $\int_0^{\pi/6} (4 \cos 2x + 3 \sec^2 x)\, dx$.

C.4 By using a double-angle identity, find the exact value of $\int_0^{\pi/4} 8 \sin^2 x\, dx$.

C.5 Use the trapezium rule with 3 intervals to estimate $\int_0^3 \sqrt{(1 + e^x)}\, dx$, giving your answer correct to 2 decimal places, and state with a reason whether your estimate is an over-estimate or an under-estimate.

C.6 The region under the curve $y = 2\cos x + \sin x$ between $x = 0$ and $x = \pi/2$ is rotated through 360° about the x-axis. Find the exact volume of the solid formed.

Chapter quiz

Q1 Find $\int e^{3x-1}\,dx$.

A	B	C	D
$(1/3)e^{3x-1} + c$	$3e^{3x-1} + c$	$e^{3x-1} + c$	$(1/3)e^{3x} + c$

Q2 Find the exact value of $\int$ from 0 to 1 of $4e^{2x}\,dx$.

A	B	C	D
$2e^2 - 2$	$2e^2$	$4e^2 - 4$	$8e^2 - 8$

Q3 Find $\int 6/(2x + 5)\,dx$.

A	B	C	D						
3 ln	2x + 5	+ c	6 ln	2x + 5	+ c	12 ln	2x + 5	+ c	$-6/(2x + 5)^2 + c$

Q4 Evaluate $\int$ from 2 to 3 of $6/(2 - 3x)\,dx$.

A	B	C	D
$2\ln(4/7)$	$2\ln(7/4)$	$6\ln(7/4)$	$-2\ln 28$

Q5 To integrate $(4x)/(2x - 1)$, we write $4x/(2x - 1) \equiv 2 + A/(2x - 1)$. What is A?

A	B	C	D
2	−2	1	−1

Q6 Find $\int 3\sin(2x + 1)\,dx$.

A	B	C	D
$-(3/2)\cos(2x + 1) + c$	$(3/2)\cos(2x + 1) + c$	$-6\cos(2x + 1) + c$	$-3\cos(2x + 1) + c$

Q7 Find $\int 4\sec^2(x/2)\,dx$.

A	B	C	D
$8\tan(x/2) + c$	$2\tan(x/2) + c$	$4\tan(x/2) + c$	$-8\tan(x/2) + c$

Q8 Which identity is used first to integrate $\int \sin^2 x\,dx$?

A	B	C	D
$\sin^2 x \equiv (1/2)(1 - \cos 2x)$	$\sin^2 x \equiv 1 - \cos^2 x$	$\sin^2 x \equiv (1/2)(1 + \cos 2x)$	$\sin 2x \equiv 2\sin x \cos x$

Q9 Find $\int \cos^2 x\,dx$.

A	B	C	D
$(1/2)x + (1/4)\sin 2x + c$	$(1/2)x - (1/4)\sin 2x + c$	$(1/2)x + (1/2)\sin 2x + c$	$(1/3)\cos^3 x + c$

Q10 Find the exact value of $\int$ from 0 to $\pi/4$ of $\tan^2 x\,dx$.

A	B	C	D
$1 - \pi/4$	$1 + \pi/4$	$\pi/4$	$2 - \pi/4$

Q11 The trapezium rule uses 2 strips on [1, 5] with $y_0 = 2$, $y_1 = 5$, $y_2 = 6$. Estimate the integral.

A	B	C	D
18	13	26	9

Q12 For a curve that bends upwards (chords lie above the curve), the trapezium rule gives:

A	B	C	D
An over-estimate	An under-estimate	The exact area	Zero error for even n

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End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/p23-5-integration/