P23·4

Differentiation

CHAPTER ASSIGNMENT · 100 MARKS

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. Give exact answers (fractions, surds, e, ln or π) unless a question asks for significant figures or decimal places. In calculus, all angles are in radians.

Practice questions from *Cambridge International AS & A Level Mathematics: Pure Mathematics 2 & 3 Coursebook* (Cambridge University Press, Sue Pemberton and Julianne Hughes), Chapter 4 (Differentiation), Exercises 4A to 4G and End-of-chapter review exercise 4, for classroom use. Where a question needs a diagram, the diagram is described in words. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Prerequisite knowledge · Check your skills

P4.1 Differentiate with respect to x: (a) $y = 5x^3 - 3x^2 + 2/x$ · (b) $y = 4x^2 - 8\sqrt{x} + 5/x^3$

P4.2 Differentiate with respect to x: (a) $(3x - 5)^4$ · (b) $1 / \sqrt{(4x^2 - 1)}$

P4.3 Find the equation of the normal to the curve $y = x^3 - 5x^2 + 2x - 1$ at the point $(1, -3)$.

P4.4 Find the stationary points on the curve $y = x^3 - 3x^2 + 2$ and determine their nature.

Exercise 4A · The product rule

4A.1 Use the product rule to differentiate each of the following with respect to x: (a) $x(x - 2)^5$ · (b) $5x(2x + 1)^3$ · (c) $x\sqrt{(x + 2)}$ · (d) $(x - 1)\sqrt{(x + 5)}$ · (e) $x^3\sqrt{(2x - 1)}$ · (f) $\sqrt{x}(x^2 + 2)^3$ · (g) $(x - 3)^2(x + 2)^5$ · (h) $(2x - 1)^5(3x + 4)^4$ · (i) $(2x - 5)(3x^2 + 1)^2$

4A.2 Find the gradient of the curve $y = x^2\sqrt{(x + 4)}$ at the point $(-3, 9)$.

4A.3 Find the equation of the tangent to the curve $y = (2 - x)^3(x + 1)^4$ at the point where $x = 1$.

4A.4 Find the gradient of the tangent to the curve $y = (x + 2)(x - 1)^3$ at the point where the curve meets the y-axis.

4A.5 Find the x-coordinate of the points on the curve $y = (3 - x)^3(x + 1)^2$ where the gradient is zero.

4A.6 Find the x-coordinate of the point on the curve $y = (x + 2)\sqrt{(1 - 2x)}$ where the gradient is zero.

4A.7 (a) Sketch the curve $y = (x - 1)^2(5 - 2x) + 3$. · (b) The curve $y = (x - 1)^2(5 - 2x) + 3$ has stationary points at A and B. The straight line through A and B cuts the axes at P and Q. Find the area of the triangle POQ.

Exercise 4B · The quotient rule

4B.1 Use the quotient rule to differentiate each of the following with respect to x: (a) $(2x + 3)/(x - 4)$ · (b) $(3x - 5)/(2 - x)$ · (c) $(x^2 - 3)/(2x - 1)$ · (d) $(3x + 1)/(2 - 5x)$ · (e) $(1 - 2x^2)/(x + 4)^2$ · (f) $5x^4/(x^2 - 1)^2$ · (g) $(3x^2 - 7x)/(x^2 + 2x + 5)$ · (h) $(x + 4)^2/(x^2 + 1)^3$

4B.2 Find the gradient of the curve $y = (x - 5)/(x + 4)$ at the point $(2, -1/2)$.

4B.3 Find the coordinates of the points on the curve $y = (x - 1)^2/(2x + 5)$ where the tangent is parallel to the x-axis.

4B.4 Find the coordinates of the points on the curve $y = (1 - 2x)/(x - 5)$ at which the gradient is 1.

4B.5 Find the equation of the tangent of the curve $y = (x - 4)/(2x + 1)$ at the point where the curve crosses the y-axis.

4B.6 Differentiate with respect to x: (a) $\sqrt{x} / (5x - 1)$ · (b) $(x - 1)/\sqrt{(2x + 3)}$ · (c) $(3 - x^2)/\sqrt{(x^2 - 1)}$ · (d) $5(x - 1)^3/\sqrt{(x + 2)}$

4B.7 Find the x-coordinate of the point on the curve $y = (x + 1)/\sqrt{(x - 1)}$ where the gradient is 0.

4B.8 Find the equation of the normal to the curve $y = (x^2 + 1)/\sqrt{(x + 2)}$ at the point $(-1, 2)$.

4B.9 The line $2x - 2y = 5$ intersects the curve $2x^2y - x^2 - 26y - 35 = 0$ at three points. (a) Find the x-coordinates of the points of intersection. · (b) Find the gradient of the curve at each of the points of intersection.

Exercise 4C · Derivatives of exponential functions

4C.1 Differentiate with respect to x: (a) e^{5x} · (b) e^{-4x} · (c) $2e^{6x}$ · (d) $3e^{-5x}$ · (e) $4e^{x/2}$ · (f) e^{2x-7} · (g) e^{x^2-3} · (h) $2x + 3e^{\sqrt{x}}$ · (i) $5\sqrt{(e^x)} - 1/e^{2x}$ · (j) $2(e^{3x} - 1)$ · (k) $(3e^{2x} + e^{-2x})/2$ · (l) $5(e^{x^2} - 2x)$

4C.2 (a) Sketch the graph of the function $y = 1 - e^{2-x}$. · (b) Find the equation of the normal to the curve $y = 1 - e^{2-x}$ at the point where $y = 0$.

4C.3 The mass, m grams, of a radioactive substance remaining t years after a given time, is given by the formula $m = 300e^{-0.00012t}$. Find the rate at which the mass is decreasing when $t = 2000$.

4C.4 Differentiate with respect to x: (a) xe^x · (b) x^2e^{3x} · (c) $5xe^{-2x}$ · (d) $2\sqrt{x}\,e^x$ · (e) e^{6x}/x · (f) $e^{-2x}/\sqrt{x}$ · (g) $(e^x - 1)/(e^x + 2)$ · (h) $xe^{3x} + e^{6x}/2$ · (i) $(x^2e^x - x)/(e^x + 2)$

4C.5 Find the gradient of the curve $y = 8/(5 + e^{2x})$ at the point where $x = 0$.

4C.6 Find the exact coordinates of the stationary point on the curve $y = xe^{x}$.

4C.7 The curve $y = 2e^{2x} + e^{-x}$ cuts the y-axis at the point P. Find the equation of the tangent to the curve at the point P and state the coordinates of the point where this tangent cuts the x-axis.

4C.8 Find the exact coordinates of the stationary point on the curve $y = (x - 4)e^{x}$ and determine its nature.

4C.9 Find the exact coordinates of the stationary point on the curve $y = e^{2x}/x^2$ and determine its nature.

4C.10 The equation of a curve is $y = x^2e^{-x}$. (a) Find the x-coordinates of the stationary points of the curve and determine the nature of these stationary points. · (b) Show that the equation of the normal to the curve at the point where $x = 1$ is $e^2x + ey = 1 + e^2$.

4C.11 Find the exact value of the x-coordinates of the points on the curve $y = x^2e^{-2x}$ at which $d^2y/dx^2 = 0$.

4C.12 Find the coordinates of the stationary point on the curve $y = e^{2x - 1}/x$.

4C.13 By writing 2 as $e^{\ln 2}$, prove that $d/dx(2^x) = 2^x \ln 2$.

4C.14 The equation of a curve is $y = x(3^x)$. Find the exact value of the gradient of the tangent to the curve at the point where $x = 1$.

Exercise 4D · Derivatives of natural logarithmic functions

4D.1 Differentiate with respect to x: (a) $\ln 3x$ · (b) $\ln 7x$ · (c) $\ln(2x + 1)$ · (d) $5 + \ln(x^2 + 1)$ · (e) $\ln(2x - 1)^2$ · (f) $\ln\sqrt{(x - 3)}$ · (g) $\ln(x + 3)^5$ · (h) $3x + \ln(2/x)$ · (i) $5x + \ln(2/(1 - 2x))$ · (j) $\ln(\ln x)$ · (k) $\ln((2 - \sqrt{x})^2)$ · (l) $\ln(5x + \ln x)$

4D.2 The answers to question 1 parts a and b are the same. Why is this the case? How many different ways can you justify this?

4D.3 Differentiate with respect to x: (a) $x \ln x$ · (b) $2x^3 \ln x$ · (c) $x \ln(2x + 1)$ · (d) $3x \ln 2x$ · (e) $x \ln(\ln x)$ · (f) $(\ln 5x)/x$ · (g) $2/(\ln x)$ · (h) $\ln(3x - 2)/x$ · (i) $\ln(2x + 1)/(4x - 1)$

4D.4 (a) Sketch the graph of the function $y = \ln(2x - 3)$. · (b) Find the gradient of the curve $y = \ln(2x - 3)$ at the point where $x = 5$.

4D.5 Find the gradient of the curve $y = e^{2x} - 5\ln(2x + 1)$ at the point where $x = 0$.

4D.6 A curve has equation $y = x^2 \ln 5x$. Find the value of dy/dx and d^2y/dx^2 at the point where $x = 2$.

4D.7 The equation of a curve is $y = x^2 \ln x$. Find the exact coordinates of the stationary point on this curve and determine whether it is a maximum or a minimum point.

4D.8 The equation of a curve is $y = (\ln x)/x$. Find the exact coordinates of the stationary point on this curve and determine whether it is a maximum or a minimum point.

4D.9 Find the equation of the tangent to the curve $y = \ln(5x - 4)$ at the point where $x = 1$.

4D.10 Use the laws of logarithms to help differentiate these expressions with respect to x: (a) $\ln\sqrt{(5x - 1)}$ · (b) $\ln(1/(3x + 2))$ · (c) $\ln[x(x + 1)^5]$ · (d) $\ln((2x + 3)/(x - 1))$ · (e) $\ln((1 - 3x)/x^2)$ · (f) $\ln[x(x - 2)/(x + 4)]$ · (g) $\ln[(3 - x)/((x + 4)(x - 1))]$ · (h) $\ln[8/((x + 1)^2(x - 2))]$ · (i) $\ln[((x + 2)(2x - 1))/(x(x + 5))]$

4D.11 Find dy/dx, in terms of x, for each of the following: (a) $e^y = 2x^2 - 1$ · (b) $e^y = 3x^3 + 2x$ · (c) $e^y = (x + 1)(x - 5)$

4D.12 A curve has equation $x = (1/5)[e^{y(2x - 3)} + 4]$. Find the value of dy/dx when x = 1.

Exercise 4E · Derivatives of trigonometric functions

4E.1 Differentiate with respect to x: (a) $2 + \sin x$ · (b) $2 \sin x + 3 \cos x$ · (c) $2 \cos x - \tan x$ · (d) $3 \sin 2x$ · (e) $4 \tan 5x$ · (f) $2 \cos 3x - \sin 2x$ · (g) $\tan(3x + 2)$ · (h) $\sin(2x + \pi/3)$ · (i) $2 \cos(3x - \pi/6)$

4E.2 Differentiate with respect to x: (a) $\sin^3 x$ · (b) $5\cos^2 3x$ · (c) $\sin^2 x - 2\cos x$ · (d) $(3 - \cos x)^4$ · (e) $2\sin^3(2x + \pi/6)$ · (f) $3\cos^4 x + 2\tan^2(2x - \pi/4)$

4E.3 Differentiate with respect to x: (a) $x \sin x$ · (b) $5x \cos 3x$ · (c) $x^2 \tan x$ · (d) $x \cos^3 2x$ · (e) $5/(\cos 3x)$ · (f) $x/(\cos x)$ · (g) $(\tan x)/x$ · (h) $(\sin x)/(2 + \cos x)$ · (i) $(\sin x)/(3x - 1)$ · (j) $1/(\sin^3 2x)$ · (k) $3x/(\sin 2x)$ · (l) $(\sin x + \cos x)/(\sin x - \cos x)$

4E.4 Differentiate with respect to x: (a) $e^{\sin x}$ · (b) $e^{\cos 2x}$ · (c) $e^{\tan 3x}$ · (d) $e^{(\sin x - \cos x)}$ · (e) $e^x \cos x$ · (f) $e^x \sin 2x$ · (g) $e^x(2 \cos x - \sin x)$ · (h) $x^3 e^{\cos x}$ · (i) $\ln(\cos x)$ · (j) $x \ln(\sin x)$ · (k) $(\cos 2x)/e^{2x} + 1$ · (l) $(x \sin 2x)/e^{2x}$

4E.5 Find the gradient of the curve $y = 3 \sin 2x - 5 \tan x$ at the point where $x = 0$.

4E.6 Find the exact value of the gradient of the curve $y = 2 \sin 3x - 4 \cos x$ at the point $(\pi/3, -2)$.

4E.7 Given that $y = \sin^2 x$ for $0 \leq x \leq \pi$, find the exact values of the x-coordinates of the points on the curve where the gradient is $\sqrt{3} / 2$.

4E.8 Prove that the gradient of the curve $y = 5/(2 - \tan x)$ is always positive.

4E.9 (a) By writing sec x as 1/(cos x), find d/dx(sec x). · (b) By writing cosec x as 1/(sin x), find d/dx(cosec x). · (c) By writing cot x as (cos x)/(sin x), find d/dx(cot x).

4E.10 Prove that the normal to the curve $y = x \sin x$ at the point $P(\pi/2, \pi/2)$ intersects the x-axis at the point $(\pi, 0)$.

4E.11 The equation of a curve is $y = 5 \sin 3x - 2 \cos x$. Find the equation of the tangent to the curve at the point $(\pi/3, -1)$. Give the answer in the form $y = mx + c$, where the values of m and c are correct to 3 significant figures.

4E.12 A curve has equation $y = 3 \cos 2x + 4 \sin 2x + 1$ for $0 \leq x \leq \pi$. Find the x-coordinates of the stationary points of the curve, giving your answer correct to 3 significant figures.

4E.13 A curve has equation $y = e^x \cos x$ for $0 \leq x \leq \pi/2$. Find the exact value of the x-coordinate of the stationary point of the curve and determine the nature of this stationary point.

4E.14 A curve has equation $y = (\sin 2x)/e^{2x}$ for $0 \leq x \leq \pi/2$. Find the exact value for the x-coordinate of the stationary point of this curve.

4E.15 A curve has equation $y = e^{3x}/(\sin 3x)$ for $0 < x < \pi/2$. Find the exact value for the x-coordinate of the stationary point of this curve and determine the nature of this point.

4E.16 A curve has equation $y = \sin 2x - x$ for $0 \leq x \leq 2\pi$. Find the x-coordinates of the stationary points of the curve, and determine the nature of these stationary points.

4E.17 A curve has equation $y = \tan x \cos 2x$ for $0 \leq x < \pi/2$. Find the x-coordinate of the stationary point on the curve giving your answer correct to 3 significant figures.

Exercise 4F · Implicit differentiation

4F.1 Differentiate each expression with respect to x: (a) y^5 · (b) $x^3 + 2y^2$ · (c) $5x^2 + \ln y$ · (d) $2 + \sin y$ · (e) $6x^2y^3$ · (f) $y^2 + xy$ · (g) $x^3 - 7xy + y^3$ · (h) $x \sin y + y \cos x$ · (i) $x^3 \ln y$ · (j) $x \cos 2y$ · (k) $5y + e^x \sin y$ · (l) $2xe^{\cos y}$

4F.2 Find dy/dx for each of these functions: (a) $x^3 + 2xy + y^3 = 10$ · (b) $x^2y + y^2 = 5x$ · (c) $2x^2 + 5xy + y^2 = 8$ · (d) $x \ln y = 2x + 5$ · (e) $2e^xy + e^{2x}y^3 = 10$ · (f) $\ln(xy) = 4 - y^2$ · (g) $xy^3 = 2 \ln y$ · (h) $\ln x - 2 \ln y + 5xy = 3$

4F.3 Find the gradient of the curve $x^2 + 3xy - 5y + y^3 = 22$ at the point (1, 3).

4F.4 Find the gradient of the curve $2x^3 - 4xy + y^3 = 16$ at the point where the curve crosses the x-axis.

4F.5 A curve has equation $2x^2 + 3y^2 - 2x + 4y = 4$. Find the equation of the tangent to the curve at the point (1, −2).

4F.6 The equation of a curve is $4x^2y + 8 \ln x + 2 \ln y = 4$. Find the equation of the normal to the curve at the point (1, 1).

4F.7 The equation of a curve is $5x^2 + 2xy + 2y^2 = 45$. (a) Given that there are two points on the curve where the tangent is parallel to the x-axis, show by differentiation that, at these points, $y = -5x$. · (b) Hence find the coordinates of the two points.

4F.8 The equation of a curve is $y^2 - 4xy - x^2 = 20$. (a) Find the coordinates of the two points on the curve where $x = 4$. · (b) Show that at one of these points the tangent to the curve is parallel to the x-axis. · (c) Find the equation of the tangent to the curve at the other point.

4F.9 The equation of a curve is $y^3 - 12xy + 16 = 0$. (a) Show that the curve has no stationary points. · (b) Find the coordinates of the point on the curve where the tangent is parallel to the y-axis.

4F.10 Find the gradient of the curve $5e^x y^2 + 2e^x y = 88$ at the point (0, 4).

4F.11 The equation of a curve is $x^2 - 4x + 6y + 2y^2 = 12$. Find the coordinates of the two points on the curve at which the gradient is 4/3.

4F.12 The equation of a curve is $2x + y \ln x = 4y$. Find the equation of the tangent to the curve at the point with coordinates (1, 1/2).

4F.13 The equation of a curve is $y = x^x$. Find the exact value of the x-coordinate of the stationary point on this curve.

4F.14 Find the stationary points on the curve $x^2 - xy + y^2 = 48$. By finding d^2y/dx^2 determine the nature of each of these stationary points.

Exercise 4G · Parametric differentiation

4G.1 For each of the following parametric equations, find dy/dx in terms of the given parameter: (a) $x = 2t^3$, $y = t^2 - 5$ · (b) $x = 2 + \sin 2\theta$, $y = 4\theta + 2\cos 2\theta$ · (c) $x = 2\theta - \sin 2\theta$, $y = 2 - \cos 2\theta$ · (d) $x = 3\tan\theta$, $y = 2\sin 2\theta$ · (e) $x = 1 + \tan\theta$, $y = \cos\theta$ · (f) $x = \cos 2\theta - \cos\theta$, $y = \sin^2\theta$ · (g) $x = 1 + 2\sin^2\theta$, $y = 4\tan\theta$ · (h) $x = 2 + e^{-t}$, $y = e^t - e^{-t}$ · (i) $x = e^{2t}$, $y = t^3e^t + 1$ · (j) $x = 2\ln(t + 3)$, $y = 4e^t$ · (k) $x = \ln(1 - t)$, $y = 5/t$ · (l) $x = 1 + \sqrt{t}$, $y = 2\ln t$

4G.2 The parametric equations of a curve are $x = 3t$, $y = t^3 + 4t^2 - 3t$. Find the two values of t for which the curve has gradient 0.

4G.3 The parametric equations of a curve are $x = 2 \sin \theta$, $y = 1 - 3 \cos 2\theta$. Find the exact gradient of the curve at the point where $\theta = \pi/3$.

4G.4 The parametric equations of a curve are $x = 2 + \ln(t - 1)$, $y = t + 4/t$ for $t > 1$. Find the coordinates of the only point on the curve at which the gradient is equal to 0.

4G.5 The parametric equations of a curve are $x = e^{2t}$, $y = 1 + 2te^{t}$. Find the equation of the normal to the curve at the point where $t = 0$.

4G.6 The parametric equations of a curve are $x = e^{2t}$, $y = t^2e^{-t} - 1$. (a) Show that $dy/dx = t(2 - t)/(2e^{3t})$. · (b) Show that the tangent to the curve at the point $(1, -1)$ is parallel to the x-axis and find the exact coordinates of the other point on the curve at which the tangent is parallel to the x-axis.

4G.7 A curve is defined by the parametric equations $x = \tan\theta$, $y = 2\sin 2\theta$, for $0 < \theta < \pi/2$. (a) Show that $dy/dx = 4\cos^2\theta(2\cos^2\theta - 1)$. · (b) Hence, find the coordinates of the stationary point.

4G.8 The parametric equations of a curve are $x = t + 4\ln t$, $y = t + 9/t$, for $t > 0$. (a) Show that $dy/dx = (t^2 - 9)/(t^2 + 4t)$. · (b) The curve has one stationary point. Find the y-coordinate of this point and determine whether it is a maximum or a minimum point.

4G.9 The parametric equations of a curve are $x = 1 + 2\sin^2\theta$, $y = 1 + 2\tan\theta$. Find the equation of the normal to the curve at the point where $\theta = \pi/4$.

4G.10 The parametric equations of a curve are $x = 2\sin\theta + \cos 2\theta$, $y = 1 + \cos 2\theta$, for $0 \le \theta \le \pi/2$. (a) Show that $dy/dx = -2\sin\theta/(2\sin\theta - 1)$. · (b) Find the coordinates of the point on the curve where the tangent is parallel to the x-axis. · (c) Show that the tangent to the curve at the point (3/2, 3/2) is parallel to the y-axis.

4G.11 The parametric equations of a curve are $x = \ln(\tan t)$, $y = 2\sin 2t$, for $0 < t < \pi/2$. (a) Show that $dy/dx = \sin 4t$. · (b) Hence show that at the point where $x = 0$ the tangent is parallel to the x-axis.

End-of-chapter review exercise 4

R4.1 The parametric equations of a curve are $x = 1 + \ln(t - 2)$, $y = t + 9/t$, for $t > 2$.

(i) Show that $dy/dx = (t^2 - 9)(t - 2)/t^2$. [3]

(ii) Find the coordinates of the only point on the curve at which the gradient is equal to 0. [3]

R4.2 Find the value of dy/dx when $x = 4$ in each of the following cases:

(i) $y = x \ln(x - 3)$, [4]

(ii) $y = (x - 1)/(x + 1)$. [3]

R4.3 The parametric equations of a curve are $x = e^{3t}$, $y = t^2e^t + 3$.

(i) Show that $dy/dx = t(t + 2)/(3e^{2t})$. [4]

(ii) Show that the tangent to the curve at the point (1, 3) is parallel to the x-axis. [2]

(iii) Find the exact coordinates of the other point on the curve at which the tangent is parallel to the x-axis. [2]

R4.4 Find the gradient of each of the following curves at the point for which $x = 0$.

(i) $y = 3 \sin x + \tan 2x$ [3]

(ii) $y = 6/(1 + e^{2x})$ [3]

R4.5 The equation of a curve is $2x^2 + 3xy + y^2 = 3$.

(i) Find the equation of the tangent to the curve at the point $(2, -1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

(ii) Show that the curve has no stationary points. [4]

R4.6 The equation of a curve is $y^3 + 4xy = 16$.

(i) Show that $dy/dx = -4y/(3y^2 + 4x)$. [4]

(ii) Show that the curve has no stationary points. [2]

(iii) Find the coordinates of the point on the curve where the tangent is parallel to the y-axis. [4]

R4.7 The equation of a curve is $y = 6 \sin x - 2 \cos 2x$. Find the equation of the tangent to the curve at the point $((1/6)\pi, 2)$. Give the answer in the form $y = mx + c$, where the values of m and c are correct to 3 significant figures. [5]

R4.8 The parametric equations of a curve are $x = 6\sin^2 t$, $y = 2\sin 2t + 3\cos 2t$, for $0 \le t < \pi$. The curve is a tilted loop in the region $x \ge 0$ that crosses the x-axis at the points D (closer to O) and B (further from O), and has stationary points at A (top) and C (bottom), as shown in the book's diagram.

(i) Show that $dy/dx = (2/3)\cot 2t - 1$. [5]

(ii) Find the values of t at A and C, giving each answer correct to 3 decimal places. [3]

(iii) Find the value of the gradient of the curve at B. [3]

R4.9 The equation of a curve is $3x^2 + 4xy + y^2 = 24$. Find the equation of the normal to the curve at the point (1, 3), giving your answer in the form $ax + by + c = 0$ where a, b and c are integers. [8]

R4.10 The diagram in the book shows the curve $y = \sqrt{((1 - x)/(1 + x))}$ for $-1 < x \leq 1$, which has a vertical asymptote at $x = -1$, crosses the y-axis at (0, 1) and meets the x-axis at (1, 0), with a point P in the first quadrant.

(i) By first differentiating $(1 - x)/(1 + x)$, obtain an expression for dy/dx in terms of x. Hence show that the gradient of the normal to the curve at the point (x, y) is $(1 + x)\sqrt{(1 - x^2)}$. [5]

(ii) The gradient of the normal to the curve has its maximum value at the point P shown in the diagram. Find, by differentiation, the x-coordinate of P. [4]

R4.11 The parametric equations of a curve are $x = 3(1 + \sin^2 t)$, $y = 2\cos^3 t$. Find dy/dx in terms of t, simplifying your answer as far as possible. [5]

R4.12 The equation of a curve is $\ln(xy) - y^3 = 1$.

(i) Show that $dy/dx = y/(x(3y^3 - 1))$. [4]

(ii) Find the coordinates of the point where the tangent to the curve is parallel to the y-axis, giving each coordinate correct to 3 significant figures. [4]

R4.13 The curve with equation $6e^{2x} + ke^y + e^{2y} = c$, where k and c are constants, passes through the point P with coordinates $(\ln 3, \ln 2)$.

(i) Show that $58 + 2k = c$. [2]

(ii) Given also that the gradient of the curve at P is -6, find the values of k and c. [5]

Check yourself

C1 Use the product rule to differentiate $y = (3x + 2)^3(2x - 1)^4$ with respect to x, giving your answer in fully factorised form.

C2 Find the exact coordinates of the stationary point on the curve $y = (2x - 1)e^{-2x}$ and determine its nature.

C3 Find the equation of the tangent to the curve $y = \ln(x^2 + 3) / x$ at the point where $x = 1$, giving your answer in exact form.

C4 A curve has equation $y = e^{2x} \sin x$ for $0 \leq x \leq \pi$. Find the exact x-coordinate of the stationary point in this interval.

C5 The equation of a curve is $x^2 + 2xy - 3y^2 = 16$. Find the equation of the normal to the curve at the point (5, 1) in the form $ax + by + c = 0$.

C6 A curve has parametric equations $x = 2t - \ln t$, $y = t^2 - 3t$ for $t > 0$. Find the exact coordinates of the stationary point on the curve.

Chapter quiz

Q1 Differentiate $y = x^2(3x - 1)^4$ with respect to x.

A	B	C	D
$2x(3x - 1)^3(9x - 1)$	$2x(3x - 1)^3(5x - 1)$	$8x(3x - 1)^3$	$2x(3x - 1)^3(3x - 1)$

Q2 Find dy/dx when $y = (2x + 3) / (x - 4)$.

A	B	C	D
$11 / (x - 4)^2$	$-11 / (x - 4)^2$	$-5 / (x - 4)^2$	$(4x - 5) / (x - 4)^2$

Q3 Find the derivative of $y = x\,e^{-5x}$ with respect to x.

A	B	C	D
$-5e^{-5x}$	$e^{-5x}(1 + 5x)$	$e^{-5x}(1 - 5x)$	$-5x\,e^{-5x}$

Q4 Find the x-coordinate of the stationary point on $y = (x - 4)e^x$.

A	B	C	D
$x = 4$	$x = -3$	$x = 0$	$x = 3$

Q5 Differentiate $y = \ln(4x - 3)$ with respect to x.

A	B	C	D
$1 / (4x - 3)$	$4 / (4x - 3)$	$4 \ln(4x - 3)$	$-3 / (4x - 3)$

Q6 Find the exact x-coordinate of the stationary point on $y = (\ln x) / x$.

A	B	C	D
$x = 1$	$x = e$	$x = 1/e$	$x = e^2$

Q7 Differentiate $y = 2 \sin 3x - 4 \cos x$ with respect to x.

A	B	C	D
$6 \cos 3x - 4 \sin x$	$2 \cos 3x + 4 \sin x$	$6 \cos 3x + 4 \sin x$	$-6 \cos 3x + 4 \sin x$

Q8 Find the gradient of $y = 3 \sin 2x - 5 \tan x$ at the point where $x = 0$.

A	B	C	D
1	−2	6	−5

Q9 Differentiate $y = (3 - 2 \cos 5x)^4$ with respect to x.

A	B	C	D
$8 \sin 5x (3 - 2 \cos 5x)^3$	$-40 \sin 5x (3 - 2 \cos 5x)^3$	$40 \sin 5x (3 - 2 \cos 5x)^3$	$40 \cos 5x (3 - 2 \cos 5x)^3$

Q10 Given $x^3 + y^3 = 4xy$, find the gradient dy/dx at the point (2, 2).

A	B	C	D
1	−1	0	−2

Q11 A curve has parametric equations $x = t^2$, $y = 5 - 2t$. Find dy/dx at $t = 2$.

A	B	C	D
$-1/2$	-2	$1/2$	$-1/4$

Q12 For $x = 2\sin\theta$, $y = 1 - 3\cos 2\theta$, find dy/dx when $\theta = \pi/3$.

A	B	C	D
$\sqrt{3}$	$6\sqrt{3}$	3	$3\sqrt{3}$

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/p23-4-differentiation/