P23·2

Logarithmic and exponential functions

CHAPTER ASSIGNMENT · 67 MARKS

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. Give exact answers unless a question asks for 3 significant figures or decimal places. Fractions are written a/b, with brackets where needed.

Practice questions from *Cambridge International AS & A Level Mathematics: Pure Mathematics 2 & 3 Coursebook* (Cambridge University Press, Sue Pemberton and Julianne Hughes), Chapter 2 (Logarithmic and exponential functions), Exercises 2A to 2H and End-of-chapter review exercise 2, for classroom use. Where a question needs a diagram, the diagram is described in words. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 2A · Logarithms to base 10

2A.1 Convert from exponential form to logarithmic form: (a) $10^2 = 100$ · (b) $10^x = 200$ · (c) $10^x = 0.05$

2A.2 Solve each of these equations, giving your answers correct to 3 significant figures: (a) $10^x = 52$ · (b) $10^x = 250$ · (c) $10^x = 0.48$

2A.3 Convert from logarithmic form to exponential form: (a) $\log_{10} 10\,000 = 4$ · (b) $\log_{10} x = 1.2$ · (c) $\log_{10} x = -0.6$

2A.4 Solve each of these equations, giving your answers correct to 3 significant figures: (a) $\log_{10} x = 1.88$ · (b) $\log_{10} x = 2.76$ · (c) $\log_{10} x = -1.4$

2A.5 Without using a calculator, find the value of: (a) $\log_{10} 100$ · (b) $\log_{10} 0.0001$ · (c) $\log_{10}(10\sqrt{10})$ · (d) $\log_{10}(\sqrt[3]{10})$ · (e) $\log_{10}(100\sqrt[3]{10})$ · (f) $\log_{10}(100 / \sqrt{1000})$

2A.6 Given that the function f is defined as $f : x \mapsto 10^x - 3$ for $x \in \mathbb{R}$, find an expression for $f^{-1}(x)$.

2A.7 Given that p and q are positive and that $4(\log_{10} p)^2 + 2(\log_{10} q)^2 = 9$, find the greatest possible value of p.

Exercise 2B · Logarithms to base a

2B.1 Convert from exponential form to logarithmic form: (a) $5^2 = 25$ · (b) $2^4 = 16$ · (c) $3^{-5} = 1/243$ · (d) $2^{-10} = 1/1024$ · (e) $8^x = 15$ · (f) $x^y = 6$ · (g) $a^b = c$ · (h) $x^{5y} = 7$

2B.2 Convert from logarithmic form to exponential form: (a) $\log_2 8 = 3$ · (b) $\log_3 81 = 4$ · (c) $\log_8 1 = 0$ · (d) $\log_{16} 4 = 1/2$ · (e) $\log_8 2 = 1/3$ · (f) $\log_2 y = 4$ · (g) $\log_a 1 = 0$ · (h) $\log_x 5 = y$

2B.3 Solve: (a) $\log_2 x = 3$ · (b) $\log_3 x = 2$ · (c) $\log_4 x = 0$ · (d) $\log_x 49 = 2$

2B.4 Solve: (a) $\log_3(x + 5) = 2$ · (b) $\log_2(3x - 1) = 5$ · (c) $\log_y(7 - 2x) = 0$

2B.5 Find the value of: (a) $\log_3 27$ · (b) $\log_6 36$ · (c) $\log_4 2$ · (d) $\log_2 0.125$ · (e) $\log_4(1/16)$ · (f) $\log_5(5\sqrt{5})$ · (g) $\log_3(\sqrt{3} / 3)$ · (h) $\log_7((\sqrt[3]{7} / 7)^2)$

2B.6 Simplify: (a) $\log_x x^3$ · (b) $\log_x \sqrt[3]{x}$ · (c) $\log_x(x^2\sqrt{x})$ · (d) $\log_x(1/x^4)$ · (e) $\log_x((1/x^3)^2)$ · (f) $\log_x(\sqrt{(x^5)})$ · (g) $\log_x(x / \sqrt[3]{x})$ · (h) $\log_x((x^2 / \sqrt{x})^3)$

2B.7 Given that the function f is defined as $f : x \mapsto 1 + \log_2(x - 3)$ for $x \in \mathbb{R}$, $x > 3$, find an expression for $f^{-1}(x)$.

2B.8 Solve: (a) $\log_2(\log_3 x) = -1$ · (b) $\log_4 2^{3-5x} = x^2$

2B.9 Without using a calculator, arrange these logarithms in ascending order: $\log_3 9$, $\log_2 8$, $\log_3 20$, $\log_3 4$, $\log_2 2$, $\log_4 3$, $\log_2 3$. Add some more logarithms of your own in the correct position in your ordered list.

Exercise 2C · The laws of logarithms

2C.1 Write as a single logarithm: (a) $\log_2 7 + \log_2 11$ · (b) $\log_6 20 - \log_6 4$ · (c) $3\log_5 2 - \log_5 4$ · (d) $2\log_3 8 - 5\log_3 2$ · (e) $1 + 2\log_2 3$ · (f) $2 - \log_4 2$

2C.2 Write as a single logarithm, then simplify your answer: (a) $\log_2 40 - \log_2 5$ · (b) $\log_6 20 + \log_6 5$ · (c) $\log_2 60 - \log_2 5$

2C.3 Simplify: (a) $3\log_5 2 + (1/2)\log_5 36 - \log_5 12$ · (b) $(1/2)\log_3 8 + (1/2)\log_3 18 - 1$

2C.4 Express 8 and 0.25 as powers of 2 and hence simplify $(\log_5 8) / (\log_5 0.25)$.

2C.5 Simplify: (a) $(\log_2 27) / (\log_2 3)$ · (b) $(\log_3 128) / (\log_3 16)$ · (c) $(\log_3 25) / (\log_3 0.04)$ · (d) $(\log_5 1000) / (\log_5 0.01)$

2C.6 Given that $\log_8 y = \log_8(x - 2) - 2\log_8 x$, express y in terms of x.

2C.7 Given that $\log_3(z - 1) - \log_3 z = 1 + 3\log_3 y$, express z in terms of y.

2C.8 Given that $y = \log_5 x$, find in terms of y: (a) x · (b) $\log_5(25x)$ · (c) $\log_5(x\sqrt{x} / 125)$ · (d) $\log_x 125$

2C.9 Given that $x = \log_4 p$ and $y = \log_4 q$, express in terms of x and/or y: (a) $\log_4(64p)$ · (b) $\log_4(2/q)$ · (c) pq · (d) $\log_4 p^2 - \log_4(4\sqrt{q})$

2C.10 Given that $\log_a x = 7$ and $\log_a y = 4$, find the value of: (a) $\log_a(x/y)$ · (b) $\log_a(\sqrt{x} / y^2)$ · (c) $\log_a(x^2\sqrt{y})$ · (d) $\log_a(y / \sqrt[3]{x})$

2C.11 A triangle has vertices labelled x (top), y (bottom-left) and z (bottom-right). The number in the rectangle on each side of the triangle is the sum of the numbers at the adjacent vertices: on side xy the sum is $-\log_2 5$; on side xz the sum is $\log_3 2 - \log_2 5$; on side yz the sum is $2\log_2 5 - 3\log_3 2$. For example, $x + y = -\log_2 5$. Find the value of x, the value of y and the value of z.

Exercise 2D · Solving logarithmic equations

2D.1 Solve: (a) $\log_5 x + \log_5 3 = \log_5 30$ · (b) $\log_3 4x - \log_3 2 = \log_3 7$ · (c) $\log_2(2 - x) + \log_2 9 = 2\log_2 8$ · (d) $\log_{10}(x - 4) = 2\log_{10} 5 + \log_{10} 2$

2D.2 Solve: (a) $\log_{10}(2x + 9) - \log_{10} 5 = 1$ · (b) $\log_3 2x - \log_3(x - 1) = 2$ · (c) $\log_2(5 - 2x) = 3 + \log_2(x + 1)$ · (d) $\log_5(2x - 3) + 2\log_5 2 = 1 + \log_5(3 - 2x)$

2D.3 Solve: (a) $\log_2 x + \log_2(x - 1) = \log_2 20$ · (b) $\log_5(x - 2) + \log_5(x - 5) = \log_5 4x$ · (c) $2 \log_3 x - \log_3(x - 2) = 2$ · (d) $3 + 2 \log_2 x = \log_2(3 - 10x)$

2D.4 Solve: (a) $\log_x 40 - \log_x 5 = 1$ · (b) $\log_x 36 + \log_x 4 = 2$ · (c) $\log_x 25 - 2 \log_x 3 = 2$ · (d) $2 \log_x 32 = 3 + 2 \log_x 4$

2D.5 Solve: (a) $(\log_2 x)^2 - 8 \log_2 x + 15 = 0$ · (b) $(\log_{10} x)^2 - \log_{10}(x^2) = 3$ · (c) $(\log_5 x)^2 + \log_5(x^3) = 10$ · (d) $4(\log_2 x)^2 - 2 \log_2(x^2) = 3$

2D.6 Solve the simultaneous equations: (a) $xy = 81$ and $\log_x y = 3$ · (b) $4^x = 2^y$ and $3 \log_{10} y = \log_{10} x + \log_{10} 2$ · (c) $\log_4(x - y) = 2 \log_4 x$ and $\log_4(x + y + 9) = 0$ · (d) $\log_{10} x = 2 \log_{10} y$ and $\log_{10}(2x - 17y) = 2$

2D.7 Given that $\log_{10}(x^2y) = 4$ and $\log_{10}(x^4 / y^3) = 18$, find the value of $\log_{10} x$ and of $\log_{10} y$.

Exercise 2E · Solving exponential equations

2E.1 Solve, giving your answers correct to 3 significant figures: (a) $5^x = 18$ · (b) $2^x = 35$ · (c) $3^{2x} = 8$ · (d) $2^{x+1} = 25$ · (e) $3^{2x-5} = 20$ · (f) $3^x = 2^{x+1}$ · (g) $5^{x+3} = 7^{4-3x}$ · (h) $4^{1-3x} = 3^{x-2}$ · (i) $2^x = 2(5^x)$ · (j) $4^x = 7(3^x)$ · (k) $2^{x+1} = 3(5^x)$ · (l) $5(2^{x-3}) = 4(3^{x+4})$

2E.2 (a) Show that $2^{x+1} + 6(2^{x-1}) = 12$ can be written as $2(2^x) + 3(2^x) = 12$. · (b) Hence solve the equation $2^{x+1} + 6(2^{x-1}) = 12$, giving your answer correct to 3 significant figures.

2E.3 Solve, giving your answers correct to 3 significant figures: (a) $2^{x+2} + 2^x = 2^2$ · (b) $3^{x+1} = 3^{x-1} + 3^2$ · (c) $2^{x+1} + 5(2^{x-1}) = 2^4$ · (d) $5^x + 5^3 = 5^{x+2}$ · (e) $4^{x-1} = 4^x - 4^3$ · (f) $3^{x+1} - 2(3^{x-2}) = 5$

2E.4 Use the substitution $y = 2^x$ to solve the equation $2^{2x} + 32 = 12(2^x)$.

2E.5 Solve, giving your answers correct to 3 significant figures: (a) $5^{2x} - 5(5^x) + 6 = 0$ · (b) $2^{2x} + 5 = 6 \times 2^x$ · (c) $3^{2x} = 6 \times 3^x + 7$ · (d) $4^{2x} + 27 = 12(4^x)$

2E.6 Use the substitution $u = 2^x$ to solve the equation $2^{2x} - 5(2^{x+1}) + 24 = 0$.

2E.7 Solve, giving your answers correct to 3 significant figures: (a) $2^{2x} - 2^{x+1} - 35 = 0$ · (b) $3^{2x} - 3^{x+1} = 10$ · (c) $2(5^{x+1}) - 5^{2x} = 16$ · (d) $4^{2x+1} = 17(4^x) - 15$

2E.8 Solve, giving your answers correct to 3 significant figures: (a) $4^x + 2^x - 12 = 0$ · (b) $3(16^x) - 10(4^x) + 3 = 0$ · (c) $4^x + 15 = 4(2^{x+1})$ · (d) $2(9^x) = 3^{x+1} + 27$

2E.9 For each of the following equations find the value of x/y correct to 3 significant figures: (a) $3^x = 7^y$ · (b) $7^x = (2.7)^y$ · (c) $4^{2x} = 3^{5y}$

2E.10 Solve the equation $x^{2.5} = 20x^{1.25}$, giving your answers in exact form.

2E.11 Solve, giving your answers correct to 3 significant figures: (a) $|4^x - 8| = 2$ · (b) $|2(3^x) - 4| = 8$ · (c) $3|2^x - 5| = 2^x$ · (d) $|3^x + 4| = |3^x - 9|$ · (e) $|5(2^x) + 3| = |5(2^x) - 10|$ · (f) $|2^{x+2} + 1| = |2^x + 12|$ · (g) $3^{2|x|} = 6(3^{|x|}) + 16$ · (h) $4^{|x|} = 5(2^{|x|}) + 14$

2E.12 Given that $2^{4x+1} \times 3^{2-x} = 8^x \times 3^{5-2x}$, find the value of: (a) 6^x · (b) x

2E.13 Solve the equation $8(8^{x-1} - 1) = 7(4^x - 2^{x+1})$.

Exercise 2F · Solving exponential inequalities

2F.1 Solve the inequalities, giving your answer in terms of base 10 logarithms: (a) $2^x < 5$ · (b) $5^x \geq 7$ · (c) $(2/3)^x > 3$ · (d) $0.8^x < 0.3$

2F.2 Solve the inequalities, giving your answer in terms of base 10 logarithms: (a) $8^{5-x} < 10$ · (b) $3^{2x+5} > 20$ · (c) $2 \times 5^{2x+1} \leq 3$ · (d) $7 \times (5/6)^{3-x} > 4$

2F.3 Solve $5^{x^2} > 2^x$, giving your answer in terms of logarithms.

2F.4 Prove that the solution to the inequality $3^{2x-1} \times 2^{1-3x} \geq 5$ is $x \geq \log(15/2) / \log(9/8)$.

2F.5 In this question you are not allowed to use a calculator. You are given that $\log_{10} 4 = 0.60206$ correct to 5 decimal places and that $10^{0.206} < 2$. (a) Find the number of digits in the number 4^{100}. · (b) Find the first digit in the number 4^{100}.

Exercise 2G · Natural logarithms

2G.1 Use a calculator to evaluate correct to 3 significant figures: (a) e^3 · (b) $e^{2.7}$ · (c) $e^{0.8}$ · (d) e^{-2}

2G.2 Use a calculator to evaluate correct to 3 significant figures: (a) $\ln 3$ · (b) $\ln 1.4$ · (c) $\ln 0.9$ · (d) $\ln 0.15$

2G.3 Without using a calculator, find the value of: (a) $e^{\ln 2}$ · (b) $e^{(1/2)\ln 9}$ · (c) $5e^{\ln 6}$ · (d) $e^{-\ln(1/2)}$

2G.4 Solve: (a) $e^{\ln x} = 5$ · (b) $\ln e^x = 15$ · (c) $e^{3\ln x} = 64$ · (d) $e^{-\ln x} = 3$

2G.5 Solve, giving your answers correct to 3 significant figures: (a) $e^{x} = 18$ · (b) $e^{2x} = 25$ · (c) $e^{x+1} = 8$ · (d) $e^{2x-3} = 16$

2G.6 Solve, giving your answers in terms of natural logarithms: (a) $e^{x} = 13$ · (b) $e^{3x} = 7$ · (c) $e^{2x-1} = 6$ · (d) $e^{(1/2)x+3} = 4$

2G.7 Solve, giving your answers in terms of natural logarithms: (a) $e^{x} > 10$ · (b) $e^{5x-2} \leq 35$ · (c) $5 \times e^{2x+3} < 1$

2G.8 Solve, giving your answers correct to 3 significant figures: (a) $\ln x = 5$ · (b) $\ln x = -4$ · (c) $\ln(x-2) = 6$ · (d) $\ln(2x+1) = -2$

2G.9 Solve, giving your answers correct to 3 significant figures: (a) $\ln(3 - x^2) = 2 \ln x$ · (b) $2 \ln(x + 2) - \ln x = \ln(2x - 1)$ · (c) $2 \ln(x + 1) = \ln(2x + 3)$ · (d) $\ln(2x + 1) = 2 \ln x + \ln 5$ · (e) $\ln(x + 2) - \ln x = 1$ · (f) $\ln(2 + x^2) = 1 + 2 \ln x$

2G.10 Express y in terms of x for each of these equations: (a) $2 \ln(y + 1) - \ln y = \ln(x + y)$ · (b) $\ln(y + 2) - \ln y = 1 + 2 \ln x$

2G.11 Solve, giving your answers in exact form: (a) $e^{2x} + 2e^x - 15 = 0$ · (b) $e^{2x} - 5e^x + 6 = 0$ · (c) $6e^{2x} - 13e^x - 5 = 0$ · (d) $e^x - 21e^{-x} = 4$

2G.12 Given that the function f is defined as $f : x \mapsto 5e^x + 2$ for $x \in \mathbb{R}$, find an expression for $f^{-1}(x)$.

2G.13 Solve $2 \ln(3 - e^{2x}) = 1$, giving your answer correct to 3 significant figures.

2G.14 Solve the simultaneous equations, giving your answers in exact form: (a) $2 \ln x + \ln y = 1 + \ln 5$ and $\ln 10x - \ln y = 2 + \ln 2$ · (b) $e^{3x + 4y} = 2e^{2x - y}$ and $e^{2x + y} = 8e^{x + 6y}$

2G.15 Solve $\ln(2x + 1) \leq \ln(x + 4)$.

Exercise 2H · Transforming a relationship to linear form

2H.1 Given that a and b are constants, use logarithms to change each of these non-linear equations into the form $Y = mX + c$. State what the variables X and Y and the constants m and c represent. (Note: there might be more than one method to do this.) (a) $y = e^{ax + b}$ · (b) $y = 10^{ax - b}$ · (c) $y = a(x^{-b})$ · (d) $y = a(b^x)$ · (e) $a = e^{x^2 + by}$ · (f) $x^a y^b = 8$ · (g) $xa^y = b$ · (h) $y = a(e^{-bx})$

2H.2 The variables x and y satisfy the equation $y = a \times x^n$, where a and n are constants. The graph of ln y (vertical axis) against ln x (horizontal axis) is a straight line passing through the points (0.31, 4.02) and (1.83, 3.22). Find the value of a and the value of n correct to 2 significant figures.

2H.3 The variables x and y satisfy the equation $y = k \times e^{n(x - 2)}$, where k and n are constants. The graph of ln y (vertical axis) against x (horizontal axis) is a straight line passing through the points (1, 1.84) and (7, 4.33). Find the value of k and the value of n correct to 2 significant figures.

2H.4 Variables x and y are related so that, when $\log_{10} y$ is plotted on the vertical axis and x is plotted on the horizontal axis, a straight-line graph passing through the points (2, 5) and (6, 11) is obtained. (a) Express $\log_{10} y$ in terms of x. · (b) Express y in terms of x, giving your answer in the form $y = a \times 10^{bx}$.

2H.5 Variables x and y are related so that, when ln y is plotted on the vertical axis and ln x is plotted on the horizontal axis, a straight-line graph passing through the points (2, 4) and (5, 13) is obtained. (a) Express ln y in terms of x. · (b) Express y in terms of x.

2H.6 The variables x and y satisfy the equation $5^{2y} = 3^{2x+1}$. By taking natural logarithms, show that the graph of ln y against ln x is a straight line, and find the exact value of the gradient of this line and state the coordinates of the point at which the line cuts the y-axis. (Note: in this equation, taking natural logarithms shows that the graph of y against x is a straight line; state its gradient and y-intercept.)

2H.7 The mass, m grams, of a radioactive substance is given by the formula $m = m_0e^{-kt}$, where t is the time in days after the mass was first recorded and m_0 and k are constants. The table below shows experimental values of t and m.

t	10	20	30	40	50
m	40.9	33.5	27.4	22.5	18.4

(a) Draw the graph of ln m against t.

(b) Use your graph to estimate the value of m_0 and k.

(c) The half-life of a radioactive substance is the time it takes to decay to half of its original mass. Find the half-life of this radioactive substance.

2H.8 The temperature, T °C, of a hot drink, t minutes after it is made, can be modelled by the equation $T = 25 + ke^{-nt}$, where k and n are constants. The table below shows experimental values of t and T.

t	2	4	6	8	10
T	63.3	57.7	52.8	48.7	45.2

(a) Convert the equation to a form suitable for drawing a straight-line graph.

(b) Draw the straight-line graph and use it to estimate the value of k and n.

(c) Estimate: (i) the initial temperature of the drink · (ii) the time taken for the temperature to reach 28 °C · (iii) the room temperature.

End-of-chapter review exercise 2

R2.1 Solve the inequality $2^x > 7$, giving your answer in terms of logarithms. [2]

R2.2 Given that $\ln p = 2 \ln q - \ln(3 + q)$ and that $q > 0$, express p in terms of q not involving logarithms. [3]

R2.3 Solve the inequality $3 \times 2^{3x + 2} < 8$, giving your answer in terms of logarithms. [4]

R2.4 Use logarithms to solve the equation $5^{x+3} = 7^{x-1}$, giving the answer correct to 3 significant figures. [4]

R2.5 Solve the equation $6(4^x) - 11(2^x) + 4 = 0$, giving your answers for x in terms of logarithms where appropriate. [5]

R2.6 Solve the equation $\ln(5x + 4) = 2 \ln x + \ln 6$. [5]

R2.7 The variables x and y satisfy the equation $y = Kx^m$, where K and m are constants. The graph of ln y (vertical axis) against ln x (horizontal axis) is a straight line passing through the points (0, 2.0) and (6, 10.2). Find the values of K and m, correct to 2 decimal places. [5]

R2.8 (i) Given that $y = 2^x$, show that the equation $2^x + 3(2^{-x}) = 4$ can be written in the form $y^2 - 4y + 3 = 0$. [3]

(ii) Hence solve the equation $2^x + 3(2^{-x}) = 4$, giving the values of x correct to 3 significant figures where appropriate. [3]

R2.9 Given that $(1.2)^x = 6^y$, use logarithms to find the value of x/y correct to 3 significant figures. [3]

R2.10 The polynomial f(x) is defined by $f(x) = 12x^3 + 25x^2 - 4x - 12$.

(i) Show that $f(-2) = 0$ and factorise f(x) completely. [4]

(ii) Given that $12 \times 27^y + 25 \times 9^y - 4 \times 3^y - 12 = 0$, state the value of 3^y and hence find y correct to 3 significant figures. [3]

R2.11 Solve the equation $|4 - 2^x| = 10$, giving your answer correct to 3 significant figures. [3]

R2.12 Use logarithms to solve the equation $e^x = 3^{x-2}$, giving your answer correct to 3 decimal places. [3]

R2.13 Using the substitution $u = 3^x$, solve the equation $3^x + 3^{2x} = 3^{3x}$ giving your answer correct to 3 significant figures. [5]

R2.14 The variables x and y satisfy the equation $5^y = 3^{2x-4}$.

(a) By taking natural logarithms, show that the graph of y against x is a straight line and find the exact value of the gradient of this line. [3]

(b) This line intersects the x-axis at P and the y-axis at Q. Find the exact coordinates of the midpoint of PQ. [3]

R2.15 The variables x and y satisfy the equation $y = K(b^x)$, where K and b are constants. The graph of ln y (vertical axis) against x (horizontal axis) is a straight line passing through the points (2.3, 1.7) and (3.1, 2.1). Find the values of K and b, correct to 2 decimal places. [6]

Check yourself

C1 Without using a calculator, evaluate $\log_2(32\sqrt{2}) - \log_3(1 / (9\sqrt{3}))$.

C2 Solve the equation $2\log_3(x + 1) - \log_3(x - 1) = 2$, showing clearly why both roots are valid or why any root is rejected.

C3 Solve the equation $3(4^x) - 5(2^x) - 2 = 0$, giving any non-integer solution in exact logarithmic form.

C4 Solve the inequality $5 \times (0.4)^{2x - 1} \geq 2$, giving your answer in terms of base-10 logarithms.

C5 Given that $f : x \mapsto 4 - 3e^{2x}$ for $x \in \mathbb{R}$, state the range of f and find an expression for $f^{-1}(x)$, stating its domain.

C6 The variables x and y satisfy $y = a \times b^{x^2}$. A graph of ln y against x^2 is a straight line passing through (1, 1.5) and (4, 3.3). Find the exact values of ln a and ln b, and hence find a and b correct to 3 significant figures.

Chapter quiz

Q1 Without a calculator, find the exact value of $\log_{10}(1000\sqrt{10})$.

A	B	C	D
2.5	3.5	1.5	4.5

Q2 Evaluate $\log_3(1/27) + \log_2(16)$ without a calculator.

A	B	C	D
1	7	−1	−7

Q3 Simplify $\log_x(\sqrt{x} / x^3)$ for $x > 0$, $x \neq 1$.

A	B	C	D
−5/2	5/2	−3/2	−2

Q4 Write $2\log_5 3 + \log_5 4 - \log_5 6$ as a single logarithm.

A	B	C	D
$\log_5 2$	$\log_5 6$	$\log_5 4$	$\log_5 18$

Q5 Given $p = \log_4 x$ and $q = \log_4 y$, express $\log_4(16x^3 / \sqrt{y})$ in terms of p and q.

A	B	C	D
$2 + 3p - \frac{1}{2}q$	$4 + 3p - \frac{1}{2}q$	$2 + 3p - 2q$	$16 + 3p - \frac{1}{2}q$

Q6 Solve $2\log_8(x + 2) = \log_8(2x + 19)$.

A	B	C	D
$x = 3$ or $x = -5$	$x = 3$ only	$x = -5$ only	$x = 5$ only

Q7 Solve $5^{2x-1} = 7$, giving the exact answer in terms of base-10 logarithms.

A	B	C	D
$x = (\lg 7 + \lg 5) / (2 \lg 5)$	$x = (\lg 7 - \lg 5) / (2 \lg 5)$	$x = \lg 35 / \lg 5$	$x = \lg(7/5) / (2 \lg 5)$

Q8 Solve $2(3^{2x}) + 7(3^x) - 15 = 0$, giving x in exact form.

A	B	C	D
$x = \ln(1.5) / \ln 3$	$x = \ln(1.5)$ or $\ln(-5)$	$x = \ln 3 / \ln 1.5$	$x = 1.5$

Q9 Solve the inequality $0.6^x < 0.7$ in terms of base-10 logarithms.

A	B	C	D
$x > \lg 0.7 / \lg 0.6$	$x < \lg 0.7 / \lg 0.6$	$x > \lg 0.6 / \lg 0.7$	$x < \lg(0.7 - 0.6)$

Q10 Without a calculator, evaluate $e^{2 \ln 5} - \ln(e^3)$.

A	B	C	D
22	7	25	19

Q11 If $y = ae^{-bx^2}$ is linearised by plotting $Y = \ln y$ against $X = x^2$, what are m and c?

A	B	C	D
$m = -b$, $c = \ln a$	$m = b$, $c = a$	$m = -bx$, $c = \ln a$	$m = \ln a$, $c = -b$

Q12 A graph of ln y against ln x is a line through (0, 2) and (4, 10) for $y = kx^n$. Find k and n.

A	B	C	D
$k = e^2$, n = 2	k = 2, n = 2	$k = e^2$, n = 2.5	$k = 10^2$, n = 2

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/p23-2-logarithmic-and-exponential-functions/