P23·1

Algebra

CHAPTER ASSIGNMENT · 73 MARKS

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. Give exact answers (fractions or surds) unless a question asks for decimal places. Always check solutions to modulus equations in the original equation.

Practice questions from *Cambridge International AS & A Level Mathematics: Pure Mathematics 2 & 3 Coursebook* (Cambridge University Press, Sue Pemberton and Julianne Hughes), Chapter 1 (Algebra), Exercises 1A to 1F and End-of-chapter review exercise 1, for classroom use. Where a question needs a graph or diagram, it is described in words. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 1A · The modulus function and modulus equations

1A.1 Solve: (a) $|4x - 3| = 7$ · (b) $|1 - 2x| = 5$ · (c) $|(3x - 2)/5| = 4$ · (d) $|x/3 + 2| = 3$ · (e) $|(x + 2)/3 - 2x/5| = 2$ · (f) $|2x + 7| = 3x$

1A.2 Solve: (a) $|(2x + 1)/(x - 2)| = 5$ · (b) $|(3x - 1)/(x + 5)| = 1$ · (c) $|2 - (x + 2)/(x - 3)| = 5$ · (d) $|3x - 5| = x + 1$ · (e) $x + |x + 4| = 8$ · (f) $8 - |1 - 2x| = x$

1A.3 Solve: (a) $|2x + 1| = |x|$ · (b) $|3 - 2x| = |3x|$ · (c) $|2x - 5| = |1 - x|$ · (d) $|3x + 5| = |1 + 2x|$ · (e) $|x - 5| = 2|x + 1|$ · (f) $3|2x - 1| = |(1/2)x - 3|$

1A.4 Solve: (a) $|x^2 - 2| = 7$ · (b) $|5 - x^2| = 3 - x$ · (c) $|x^2 + 2x| = x + 2$ · (d) $|x^2 - 3| = 2x + 1$ · (e) $|2x^2 - 5x| = 4 - x$ · (f) $|x^2 - 7x + 6| = 6 - x$

1A.5 Solve the simultaneous equations: (a) $x + 2y = 8$ and $|x + 2| + y = 6$ · (b) $3x + y = 0$ and $y = |2x^2 - 5|$

1A.6 Solve the equation $5|x - 1|^2 + 9|x - 1| - 2 = 0$.

1A.7 (a) Solve the equation $x^2 - 5|x| + 6 = 0$. (b) Sketch the graph of $y = x^2 - 5|x| + 6$ (in the book: use graphing software to draw the graph). (c) Name the equation of the line of symmetry of the curve.

1A.8 Solve the equation $|2x + 1| + |2x - 1| = 3$.

1A.9 Solve the equation $|3x - 2y - 11| + 2\sqrt{(31 - 8x + 5y)} = 0$.

Exercise 1B · Graphs of y = |f(x)| where f(x) is linear

1B.1 Sketch the graphs of each of the following functions showing the coordinates of the points where the graph meets the axes. Express each function in an alternative form: (a) $y = |x + 2|$ · (b) $y = |3 - x|$ · (c) $y = |5 - (1/2)x|$

1B.2 (a) Complete the table of values for $y = |x - 3| + 2$. (b) Draw the graph of $y = |x - 3| + 2$ for $0 \le x \le 6$. (c) Describe the transformation that maps the graph of $y = |x|$ onto the graph of $y = |x - 3| + 2$.

x	0	1	2	3	4	5	6
y	5		3				

1B.3 Describe fully the transformation (or combination of transformations) that maps the graph of $y = |x|$ onto each of these functions: (a) $y = |x + 1| + 2$ · (b) $y = |x - 5| - 2$ · (c) $y = 2 - |x|$ · (d) $y = |2x| - 3$ · (e) $y = 1 - |x + 2|$ · (f) $y = 5 - 2|x|$

1B.4 Sketch the graphs of each of the functions in question 3. For each graph, state the coordinates of the vertex.

1B.5 $f(x) = |5 - 2x| + 3$ for $2 \leq x \leq 8$. Find the range of function f.

1B.6 (a) Sketch the graph of $y = 2|x - 2| + 1$ for $-2 < x < 6$, showing the coordinates of the vertex and the y-intercept. (b) On the same diagram, sketch the graph of $y = x + 2$. (c) Use your graph to solve the equation $2|x - 2| + 1 = x + 2$.

1B.7 (a) Sketch the graph of $y = |x - 2|$ for $-3 < x < 6$, showing the coordinates of the vertex and the y-intercept. (b) On the same diagram, sketch the graph of $y = |1 - 2x|$. (c) Use your graph to solve the equation $|x - 2| = |1 - 2x|$.

1B.8 (a) Sketch the graph of $y = |x + 1| + |x - 1|$. (b) Use your graph to solve the equation $|x + 1| + |x - 1| = 4$.

Exercise 1C · Solving modulus inequalities

1C.1 The graphs of $y = |x - 1|$ and $y = 2|x - 4|$ are shown on a grid for $-1 \le x \le 9$ and $0 \le y \le 8$ (the V-shaped graph $y = |x - 1|$ has vertex at (1, 0), the steeper V-shaped graph $y = 2|x - 4|$ has vertex at (4, 0), and the two graphs intersect at (3, 2) and (7, 6)). Write down the solution to the inequality $|x - 1| > 2|x - 4|$.

1C.2 (a) On the same axes, sketch the graphs of $y = |2x - 1|$ and $y = 4 - |x - 1|$. (b) Solve the inequality $|2x - 1| > 4 - |x - 1|$.

1C.3 Solve (you may use either an algebraic method or a graphical method): (a) $|2x + 1| \le 3$ · (b) $|2 - x| < 4$ · (c) $|3x - 2| \ge 7$

1C.4 Solve (you may use either an algebraic method or a graphical method): (a) $|2x - 5| \leq x - 2$ · (b) $|3 + x| > 4 - 2x$ · (c) $|x - 7| - 2x \leq 4$

1C.5 Solve (you may use either an algebraic method or a graphical method): (a) $|x + 4| < |2x|$ · (b) $|2x - 5| > |3 - x|$ · (c) $|3x - 2| \leq |1 - 3x|$ · (d) $|x - 1| > 2|x - 4|$ · (e) $3|3 - x| \geq |2x - 1|$ · (f) $|3x - 5| < 2|2 - x|$

1C.6 Solve $|2x + 1| + |2x - 1| > 5$.

Exercise 1D · Division of polynomials

1D.1 Simplify each of the following: (a) $(x^3 + 5x^2 + 5x - 2) \div (x + 2)$ · (b) $(x^3 - 8x^2 + 22x - 21) \div (x - 3)$ · (c) $(3x^3 - 7x^2 + 6x - 2) \div (x - 1)$ · (d) $(2x^3 - 3x^2 + 8x + 5) \div (2x + 1)$ · (e) $(5x^3 - 13x^2 + 10x - 8) \div (2 - x)$ · (f) $(6x^4 + 13x - 19) \div (1 - x)$

1D.2 Find the quotient and remainder for each of the following: (a) $(x^3 + 2x^2 + 5x - 4) \div (x + 1)$ · (b) $(6x^3 + 7x^2 - 6) \div (x - 2)$ · (c) $(8x^3 + x^2 - 2x + 1) \div (2x - 1)$ · (d) $(2x^3 - 9x^2 - 9) \div (3 - x)$ · (e) $(x^3 + 5x^2 - 3x - 1) \div (x^2 + 2x + 1)$ · (f) $(5x^4 - 2x^2 - 13x + 8) \div (x^2 + 1)$

1D.3 Use algebraic division to show these two important factorisations: (a) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ · (b) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

1D.4 (a) Use algebraic division to show that $x - 2$ is a factor of $2x^3 + 7x^2 - 43x + 42$. (b) Hence, factorise $2x^3 + 7x^2 - 43x + 42$ completely.

1D.5 (a) Use algebraic division to show that $2x + 1$ is a factor of $12x^3 + 16x^2 - 3x - 4$. (b) Hence, solve the equation $12x^3 + 16x^2 - 3x - 4 = 0$.

1D.6 (a) Use algebraic division to show that $x + 1$ is a factor of $x^3 - x^2 + 2x + 4$. (b) Hence, show that there is only one real root for the equation $x^3 - x^2 + 2x + 4 = 0$.

1D.7 The general formula for solving the quadratic equation $ax^2 + bx + c = 0$ is $x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$. There is a general formula for solving a cubic equation. Find out more about this formula and use it to solve $2x^3 - 5x^2 - 28x + 15 = 0$.

Exercise 1E · The factor theorem

1E.1 Use the factor theorem to show that $x + 1$ is a factor of $x^4 - 3x^3 - 4x^2 + 5x + 5$.

1E.2 Use the factor theorem to show that $2x - 5$ is a factor of $2x^3 - 7x^2 + 9x - 10$.

1E.3 Given that $x + 4$ is a factor of $x^3 + ax^2 - 29x + 12$, find the value of a.

1E.4 Given that $x - 3$ is a factor of $x^3 + ax^2 + bx - 30$, express a in terms of b.

1E.5 Given that $2x^2 + x - 1$ is a factor of $2x^3 - x^2 + ax + b$, find the value of a and the value of b.

1E.6 Given that $x - 3$ and $2x + 1$ are factors of $2x^3 + px^2 + (2q - 1)x + q$: (a) Find the value of p and the value of q. (b) Explain why $x + 2$ is also a factor of the expression.

1E.7 It is given that $x + a$ is a factor of $x^3 + 4x^2 + 7ax + 4a$. Find the possible values of a.

1E.8 (a) It is given that $x + 1$ is a common factor of $x^3 + px + q$ and $x^3 + (1 - p)x^2 + 19x - 2q$. Find the value of p and the value of q. (b) When p and q have these values, factorise $x^3 + px + q$ and $x^3 + (1 - p)x^2 + 19x - 2q$ completely.

1E.9 Given that $x - 1$ and $x + 2$ are factors of $x^4 - x^3 + px^2 - 11x + q$: (a) Find the value of p and the value of q. (b) When p and q have these values, factorise $x^4 - x^3 + px^2 - 11x + q$ completely.

1E.10 Solve: (a) $x^3 - 5x^2 - 4x + 20 = 0$ · (b) $x^3 + 5x^2 - 17x - 21 = 0$ · (c) $2x^3 - 5x^2 - 13x + 30 = 0$ · (d) $3x^3 + 17x^2 + 18x - 8 = 0$ · (e) $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$ · (f) $2x^4 - 11x^3 + 12x^2 + x - 4 = 0$

1E.11 You are given that the equation $x^3 + ax^2 + bx + c = 0$ has three real roots and that these roots are consecutive terms in an arithmetic progression. Show that $2a^3 + 27c = 9ab$.

1E.12 Find the set of values for k for which the equation $3x^4 + 4x^3 - 12x^2 + k = 0$ has four real roots.

Exercise 1F · The remainder theorem

1F.1 Find the remainder when: (a) $6x^3 + 3x^2 - 5x + 2$ is divided by $x - 1$ · (b) $x^3 + x^2 - 11x + 12$ is divided by $x + 4$ · (c) $x^4 + 2x^3 - 5x^2 - 2x + 8$ is divided by $x + 1$ · (d) $4x^3 - x^2 - 18x + 1$ is divided by $2x - 1$

1F.2 (a) When $x^3 - 3x^2 + ax - 7$ is divided by $x + 2$, the remainder is -37. Find the value of a. (b) When $9x^3 + bx - 5$ is divided by $3x + 2$, the remainder is -13. Find the value of b.

1F.3 $f(x) = x^3 + ax^2 + bx - 5$. $f(x)$ has a factor of $x - 1$ and leaves a remainder of -6 when divided by $x + 1$. Find the value of a and the value of b.

1F.4 The polynomial $3x^3 + ax^2 + bx + 8$, where a and b are constants, is denoted by $f(x)$. It is given that $x + 2$ is a factor of $f(x)$, and that when $f(x)$ is divided by $x - 1$ the remainder is 15. Find the value of a and the value of b.

1F.5 The function $f(x) = ax^3 + 7x^2 + bx - 8$, where a and b are constants, is such that $2x + 1$ is a factor. The remainder when $f(x)$ is divided by $x + 1$ is 7. (a) Find the value of a and the value of b. (b) Factorise $f(x)$ completely.

1F.6 The polynomial $x^3 + 6x^2 + px - 3$ leaves a remainder of R when divided by $x + 1$ and a remainder of $-10R$ when divided by $x - 3$. (a) Find the value of p. (b) Hence find the remainder when the expression is divided by $x - 2$.

1F.7 The polynomial $x^3 + ax^2 + bx + 2$, where a and b are constants, is denoted by f(x). It is given that $x - 2$ is a factor of f(x), and that when f(x) is divided by $x + 1$ the remainder is 21. (a) Find the value of a and the value of b. (b) Solve the equation $f(x) = 0$, giving the roots in exact form.

1F.8 The polynomial $2x^3 + ax^2 + bx + c$ is denoted by f(x). The roots of $f(x) = 0$ are -1, 2 and k. When f(x) is divided by $x - 1$, the remainder is 6. (a) Find the value of k. (b) Find the remainder when f(x) is divided by $x + 2$.

1F.9 $P(x) = 2x + 4x^2 + 6x^3 + \ldots + 100x^{50}$. Find the remainder when P(x) is divided by $(x - 1)$.

1F.10 $P(x) = 3(x + 1)(x + 2)(x + 3) + a(x + 1)(x + 2) + b(x + 1) + c$. It is given that when $P(x)$ is divided by each of $x + 1$, $x + 2$ and $x + 3$ the remainders are -2, 2 and 10, respectively. Find the value of a, the value of b and the value of c.

End-of-chapter review exercise 1

R1.1 Solve the equation $|2x - 3| = |5x + 1|$. [3]

R1.2 Solve the inequality $|5x - 3| \geq 7$. [3]

R1.3 Solve the inequality $|2x - 3| < |2 - x|$. [3]

R1.4 Solve the equation $|x^2 - 14| = 11$. [4]

R1.5 The polynomial $ax^3 - 13x^2 - 41x - 2a$, where a is a constant, is denoted by p(x). (a) Given that (x − 4) is a factor of p(x), find the value of a. [2] (b) When a has this value, factorise p(x) completely. [3]

R1.6 The polynomial $6x^3 - 23x^2 - 38x + 15$ is denoted by f(x). (a) Show that (x − 5) is a factor of f(x) and hence factorise f(x) completely. [4] (b) Write down the roots of $f(|x|) = 0$. [1]

R1.7 The polynomial $x^3 - 5x^2 + ax + b$ is denoted by f(x). It is given that (x + 2) is a factor of f(x) and that when f(x) is divided by (x − 1) the remainder is −6. Find the value of a and the value of b. [5]

R1.8 The polynomial $x^3 - 5x^2 + 7x - 3$ is denoted by p(x). (a) Find the quotient and remainder when p(x) is divided by $(x^2 - 2x - 1)$. [4] (b) Use the factor theorem to show that (x − 3) is a factor of p(x). [2]

R1.9 The polynomial $4x^4 + 4x^3 - 7x^2 - 4x + 8$ is denoted by p(x). (a) Find the quotient and remainder when p(x) is divided by $(x^2 - 1)$. [3] (b) Hence solve the equation $4x^4 + 4x^3 - 7x^2 - 4x + 3 = 0$. [3]

R1.10 The polynomial $x^4 - 48x^2 - 21x - 2$ is denoted by f(x). (a) Find the value of the constant k for which $f(x) = (x^2 + kx + 2)(x^2 - kx - 1)$. [3] (b) Hence solve the equation f(x) = 0. Give your answers in exact form. [3]

R1.11 The polynomial $2x^4 + 3x^3 - 12x^2 - 7x + a$ is denoted by p(x). (a) Given that (2x – 1) is a factor of p(x), find the value of a. [2] (b) When a has this value, verify that (x + 3) is also a factor of p(x) and hence factorise p(x) completely. [4]

R1.12 The polynomial $3x^3 + ax^2 - 36x + 20$ is denoted by p(x). (a) Given that (x – 2) is a factor of p(x), find the value of a. [2] (b) When a has this value, solve the equation p(x) = 0. [4]

R1.13 The polynomial $2x^3 + 5x^2 - 7x + 11$ is denoted by f(x). (a) Find the remainder when f(x) is divided by (x – 2). [2] (b) Find the quotient and remainder when f(x) is divided by $(x^2 - 4x + 2)$. [4]

R1.14 The polynomial $ax^3 + bx^2 - x + 12$ is denoted by p(x). (a) Given that (x − 3) and (x + 1) are factors of p(x), find the value of a and the value of b. [4] (b) When a and b take these values, find the other linear factor of p(x). [2]

R1.15 The polynomial $6x^3 + x^2 + ax - 10$, where a is a constant, is denoted by P(x). It is given that when P(x) is divided by (x + 2) the remainder is −12. (a) Find the value of a and hence verify that (2x + 1) is a factor of P(x). [3] (b) When a has this value, solve the equation P(x) = 0. [4]

R1.16 The polynomial $2x^3 + ax^2 + bx + 6$ is denoted by p(x). (a) Given that (x + 2) and (x − 3) are factors of p(x), find the value of a and the value of b. [4] (b) When a and b take these values, factorise p(x) completely. [3]

R1.17 The polynomials P(x) and Q(x) are defined as: $P(x) = x^3 + ax^2 + b$ and $Q(x) = x^3 + bx^2 + a$. It is given that $(x - 2)$ is a factor of P(x) and that when Q(x) is divided by $(x + 1)$ the remainder is -15. (a) Find the value of a and the value of b. [5] (b) When a and b take these values, find the least possible value of $P(x) - Q(x)$ as x varies. [2]

R1.18 The polynomial $5x^3 - 13x^2 + 17x - 7$ is denoted by p(x). (a) Find the quotient when p(x) is divided by $(x - 1)$, and show that the remainder is 2. [4] (b) Hence show that the polynomial $5x^3 - 13x^2 + 17x - 9$ has exactly one real root. [3]

R1.19 The polynomial $4x^3 + kx^2 - 65x + 18$ is denoted by f(x). (a) Given that $(x + 2)$ is a factor of f(x), find the value of k. [2] (b) When k has this value, solve the equation $f(x) = 0$. [4] (c) Write down the roots of $f(x^2) = 0$. [1]

R1.20 The polynomial $2x^3 - 5x^2 + ax + b$, where a and b are constants, is denoted by f(x). It is given that when f(x) is divided by (x + 2) the remainder is 8 and that when f(x) is divided by (x − 1) the remainder is 50. (a) Find the value of a and the value of b. [5] (b) When a and b have these values, find the quotient and remainder when f(x) is divided by $x^2 - x + 2$. [3]

R1.21 The polynomial $2x^3 - 9x^2 + ax + b$, where a and b are constants, is denoted by f(x). It is given that (x + 2) is a factor of f(x), and that when f(x) is divided by (x + 1) the remainder is 30. (a) Find the value of a and the value of b. [5] (b) When a and b have these values, solve the equation f(x) = 0. [4]

R1.22 The polynomial $x^3 + 3x^2 + 4x + 2$ is denoted by f(x). (i) Find the quotient and remainder when f(x) is divided by $x^2 + x - 1$. [4] (ii) Use the factor theorem to show that (x + 1) is a factor of f(x). [2]

R1.23 The polynomial $4x^3 + ax^2 + 9x + 9$, where a is a constant, is denoted by p(x). It is given that when p(x) is divided by (2x – 1) the remainder is 10. (i) Find the value of a and hence verify that (x – 3) is a factor of p(x). [3] (ii) When a has this value, solve the equation p(x) = 0. [4]

R1.24 The polynomial $ax^3 – 5x^2 + bx + 9$, where a and b are constants, is denoted by p(x). It is given that (2x + 3) is a factor of p(x), and that when p(x) is divided by (x + 1) the remainder is 8. (i) Find the values of a and b. [5] (ii) When a and b have these values, factorise p(x) completely. [3]

Check yourself

C1 Solve the equation $|3x – 4| = x + 2$, checking both candidate roots in the original equation.

C2 Sketch the graph of $y = 3 - |2x - 1|$, labelling the coordinates of its vertex and its intercepts with both axes.

C3 Solve the inequality $|2x + 3| > |x - 3|$.

C4 Find the quotient and remainder when $3x^3 - 4x^2 + 2x - 5$ is divided by $x - 2$.

C5 Given that $(x + 3)$ is a factor of $2x^3 + 5x^2 - 4x + k$, find k and hence factorise the cubic completely.

C6 When $f(x) = 2x^3 + px^2 + qx + 6$ is divided by $(x - 1)$ the remainder is 12, and $(x + 2)$ is a factor of $f(x)$. Find the values of p and q.

Chapter quiz

Q1 Solve $|3x - 2| = 7$.

A	B	C	D
$x = 3$ only	$x = 3$ or $x = -5/3$	$x = -3$ or $x = 5/3$	$x = 3$ or $x = 5/3$

Q2 Solve $|x - 5| = 2x + 1$.

A	B	C	D
$x = 4/3$ only	$x = -6$ or $x = 4/3$	$x = -6$ only	$x = 2$ or $x = -4/3$

Q3 What are the coordinates of the vertex of $y = 2|x - 3| - 5$?

A	B	C	D
$(-3, -5)$	$(3, 5)$	$(3, -5)$	$(2, -3)$

Q4 How is the graph of $y = f(|x|)$ obtained from the graph of $y = f(x)$?

A	B	C	D		
Reflect the part below the x-axis in the x-axis	Keep the part for $x \geq 0$ and reflect it in the y-axis for $x < 0$	Reflect the whole graph in the line $y = x$	Translate the graph left by	x	units

Q5 Solve the inequality $|2x + 3| < 7$.

A	B	C	D
$x < -5$ or $x > 2$	$-2 < x < 5$	$-5 < x < 2$	$-7 < x < 2$

Q6 Solve the inequality $|x + 2| \geq |2x - 1|$.

A	B	C	D
$-1/3 \leq x \leq 3$	$x \leq -1/3$ or $x \geq 3$	$-3 \leq x \leq 1/3$	$x \geq -1/3$

Q7 Find the quotient when $x^3 + 4x^2 - 7x - 10$ is divided by $x - 2$.

A	B	C	D
$x^2 + 2x - 3$	$x^2 + 6x + 5$	$x^2 - 6x + 5$	$x^2 + 4x + 5$

Q8 Find the remainder when $2x^3 - 3x^2 + 4x - 5$ is divided by $x + 2$.

A	B	C	D
-1	-17	-41	11

Q9 Given that $x - 3$ is a factor of $x^3 + kx^2 - 5x + 6$, find the value of k.

A	B	C	D
$k = -2$	$k = 2$	$k = -4$	$k = 6$

Q10 Find the remainder when $4x^3 - 4x^2 + 3x + 2$ is divided by $2x - 1$.

A	B	C	D
1	2	3	-1

Q11 When a polynomial $P(x)$ is divided by a quadratic $x^2 - 1$, the remainder $R(x)$ has degree:

A	B	C	D
At most 1 (linear or constant)	Always 2	Always 0 (a constant only)	Equal to the degree of P(x)

Q12 Solve the cubic equation $x^3 - 2x^2 - 5x + 6 = 0$.

A	B	C	D
x = −1, 2 or −3	x = −2, 1 or 3	x = 1, 2 or 3	x = −2, −1 or 6

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/p23-1-algebra/