P1·7

Trigonometry

CHAPTER ASSIGNMENT

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. Give exact answers (fractions, surds, multiples of π) where appropriate, or round angles in degrees to 1 decimal place and angles in radians to 3 significant figures unless a question specifies otherwise. Fractions are written a/b, with brackets where needed.

Practice questions from *Pure Mathematics 1* (Hodder Education, Sophie Goldie), Chapter 7 (Trigonometry), Exercises 7A to 7F, for classroom use. Where a question needs a diagram, the diagram is described in words. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 7A · Right-angled triangles and exact values

7A.1 In the triangle PQR, PQ = 17 cm, QR = 15 cm and PR = 8 cm. (i) Show that the triangle is right-angled. (ii) Write down the values of sin Q, cos Q and tan Q, leaving your answers as fractions. (iii) Use your answers to part (ii) to show that (a) $\sin^2 Q + \cos^2 Q = 1$ · (b) $\tan Q = (\sin Q)/(\cos Q)$.

7A.2 Without using a calculator, show that: (i) $\sin 60° \cos 30° + \cos 60° \sin 30° = 1$ · (ii) $\sin^2 30° + \sin^2 45° = \sin^2 60°$ · (iii) $3 \sin^2 30° = \cos^2 30°$.

7A.3 In the diagram, AB = 10 cm, angle BAC = 30°, angle BCD = 45° and angle BDC = 90°. (In the diagram, A, C and D lie in that order along a horizontal straight line, and B is vertically above D.) (i) Find the length of BD. (ii) Show that AC = $5(\sqrt{3} - 1)$ cm.

7A.4 In the diagram, OA = 1 cm, angle AOB = angle BOC = angle COD = 30° and angle OAB = angle OBC = angle OCD = 90°. (Three right-angled triangles OAB, OBC and OCD are joined in a fan around O.) (i) Find the length of OD giving your answer in the form $a\sqrt{3}$. (ii) Show that the perimeter of OABCD is $(5/3)(1 + \sqrt{3})$ cm.

7A.5 In the diagram, ABED is a trapezium with right angles at E and D, and CED is a straight line. The lengths of AB and BC are 2d and $(2\sqrt{3})$d respectively, and angles BAD and CBE are 30° and 60° respectively. (In the diagram, AD and BE are horizontal, CED is vertical with E between C and D, and B lies between A and C.) (i) Find the length of CD in terms of d. (ii) Show that angle CAD = $\tan^{-1}(2/\sqrt{3})$. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q3 November 2005]

7A.6 In the diagram, ABC is a triangle in which AB = 4 cm, BC = 6 cm and angle ABC = 150°. The line CX is perpendicular to the line ABX (where A, B and X lie in that order along a horizontal straight line). (i) Find the exact length of BX and show that angle CAB = $\tan^{-1}(3/(4 + 3\sqrt{3}))$. (ii) Show that the exact length of AC is $\sqrt{(52 + 24\sqrt{3})}$ cm. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q6 June 2006]

Exercise 7B · Trigonometrical identities

7B.1 Prove the identity $1 - \cos^2 \theta \equiv \sin^2 \theta$.

7B.2 Prove the identity $(1 - \sin^2 \theta) \tan \theta \equiv \cos \theta \sin \theta$.

7B.3 Prove the identity $1/\sin^2 \theta - (\cos^2 \theta)/(\sin^2 \theta) \equiv 1$.

7B.4 Prove the identity $\tan^2 \theta \equiv 1/\cos^2 \theta - 1$.

7B.5 Prove the identity $(\sin^2 \theta - 3\cos^2 \theta + 1)/(\sin^2 \theta - \cos^2 \theta) \equiv 2$.

7B.6 Prove the identity $1/\cos^2 \theta + 1/\sin^2 \theta \equiv 1/(\cos^2 \theta \sin^2 \theta)$.

7B.7 Prove the identity $\tan \theta + (\cos \theta)/(\sin \theta) \equiv 1/(\sin \theta \cos \theta)$.

7B.8 Prove the identity $1/(1 + \sin \theta) + 1/(1 - \sin \theta) \equiv 2/\cos^2 \theta$.

7B.9 Prove the identity $(1 - \tan^2 x)/(1 + \tan^2 x) \equiv 1 - 2\sin^2 x$. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q3 June 2007]

7B.10 Prove the identity $(1 + \sin x)/(\cos x) + (\cos x)/(1 + \sin x) \equiv 2/(\cos x)$. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q2 November 2008]

7B.11 Prove the identity $(\sin x)/(1 - \sin x) - (\sin x)/(1 + \sin x) \equiv 2\tan^2 x$. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q1 June 2009]

Exercise 7C · Graphs and trigonometrical equations in degrees

7C.1 (i) Sketch the curve $y = \sin x$ for $0° \le x \le 360°$. (ii) Solve the equation $\sin x = 0.5$ for $0° \le x \le 360°$, and illustrate the two roots on your sketch. (iii) State the other roots for $\sin x = 0.5$, given that x is no longer restricted to values between 0° and 360°. (iv) Write down, without using your calculator, the value of $\sin 330°$.

7C.2 (i) Sketch the curve $y = \cos x$ for $-90° \leq x \leq 450°$. (ii) Solve the equation $\cos x = 0.6$ for $-90° \leq x \leq 450°$, and illustrate all the roots on your sketch. (iii) Sketch the curve $y = \sin x$ for $-90° \leq x \leq 450°$. (iv) Solve the equation $\sin x = 0.8$ for $-90° \leq x \leq 450°$, and illustrate all the roots on your sketch. (v) Explain why some of the roots of $\cos x = 0.6$ are the same as those for $\sin x = 0.8$, and why some are different.

7C.3 Solve the following equations for $0° \leq x \leq 360°$: (i) $\tan x = 1$ · (ii) $\cos x = 0.5$ · (iii) $\sin x = -\sqrt{3}/2$ · (iv) $\tan x = -1$ · (v) $\cos x = -0.9$ · (vi) $\cos x = 0.2$ · (vii) $\sin x = -0.25$ · (viii) $\cos x = -1$.

7C.4 Write the following as integers, fractions, or using square roots. You should not need your calculator: (i) $\sin 60°$ · (ii) $\cos 45°$ · (iii) $\tan 45°$ · (iv) $\sin 150°$ · (v) $\cos 120°$ · (vi) $\tan 180°$ · (vii) $\sin 390°$ · (viii) $\cos(-30°)$ · (ix) $\tan 315°$.

7C.5 In this question all the angles are in the interval $-180°$ to $180°$. Give all answers correct to 1 decimal place. (i) Given that $\sin\alpha < 0$ and $\cos\alpha = 0.5$, find α. (ii) Given that $\tan\beta = 0.4463$ and $\cos\beta < 0$, find β. (iii) Given that $\sin\gamma = 0.8090$ and $\tan\gamma > 0$, find γ.

7C.6 (i) Draw a sketch of the graph $y = \sin x$ and use it to demonstrate why $\sin x = \sin(180° - x)$. (ii) By referring to the graphs of $y = \cos x$ and $y = \tan x$, state whether the following are true or false: (a) $\cos x = \cos(180° - x)$ · (b) $\cos x = -\cos(180° - x)$ · (c) $\tan x = \tan(180° - x)$ · (d) $\tan x = -\tan(180° - x)$.

7C.7 (i) For what values of α are $\sin\alpha$, $\cos\alpha$ and $\tan\alpha$ all positive? (ii) Are there any values of α for which $\sin\alpha$, $\cos\alpha$ and $\tan\alpha$ are all negative? Explain your answer. (iii) Are there any values of α for which $\sin\alpha$, $\cos\alpha$ and $\tan\alpha$ are all equal? Explain your answer.

7C.8 Solve the following equations for $0° \le x \le 360°$: (i) $\sin x = 0.1$ · (ii) $\cos x = 0.5$ · (iii) $\tan x = -2$ · (iv) $\sin x = -0.4$ · (v) $\sin^2 x = 1 - \cos x$ · (vi) $\sin^2 x = 1$ · (vii) $1 - \cos^2 x = 2 \sin x$ · (viii) $\sin^2 x = 2 \cos^2 x$ · (ix) $2 \sin^2 x = 3 \cos x$ · (x) $3 \tan^2 x - 10 \tan x + 3 = 0$.

7C.9 The diagram shows part of the curves $y = \cos x°$ and $y = \tan x°$ for $0° \leq x \leq 180°$ (with an asymptote at $x = 90°$), which intersect at the points A (in $0° < x < 90°$) and B (in $90° < x < 180°$). Find the co-ordinates of A and B.

7C.10 (i) Show that the equation $3(2 \sin x - \cos x) = 2(\sin x - 3 \cos x)$ can be written in the form $\tan x = -3/4$. (ii) Solve the equation $3(2 \sin x - \cos x) = 2(\sin x - 3 \cos x)$, for $0° \leq x \leq 360°$. [Cambridge AS & A Level Mathematics 9709, Paper 12 Q1 June 2010]

7C.11 (i) Prove the identity $(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$. (ii) Solve the equation $(\sin x + \cos x)(1 - \sin x \cos x) = 9 \sin^3 x$ for $0° \leq x \leq 360°$. [Cambridge AS & A Level Mathematics 9709, Paper 12 Q5 November 2009]

7C.12 (i) Show that the equation $\sin\theta + \cos\theta = 2(\sin\theta - \cos\theta)$ can be expressed as $\tan\theta = 3$. (ii) Hence solve the equation $\sin\theta + \cos\theta = 2(\sin\theta - \cos\theta)$, for $0° \le \theta \le 360°$. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q3 June 2005]

7C.13 Solve the equation $3\sin^2\theta - 2\cos\theta - 3 = 0$, for $0° \le \theta \le 180°$. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q1 November 2005]

Exercise 7D · Circular measure (radians)

7D.1 Express the following angles in radians, leaving your answers in terms of π where appropriate: (i) 45° · (ii) 90° · (iii) 120° · (iv) 75° · (v) 300° · (vi) 23° · (vii) 450° · (viii) 209° · (ix) 150° · (x) 7.2°.

7D.2 Express the following angles in degrees, using a suitable approximation where necessary: (i) $\pi/10$ · (ii) $3\pi/5$ · (iii) 2 radians · (iv) $4\pi/9$ · (v) 3π · (vi) $5\pi/3$ · (vii) 0.4 radians · (viii) $3\pi/4$ · (ix) $7\pi/3$ · (x) $3\pi/7$.

7D.3 Write the following as fractions, or using square roots. You should not need your calculator: (i) $\sin(\pi/4)$ · (ii) $\tan(\pi/3)$ · (iii) $\cos(\pi/6)$ · (iv) $\cos\pi$ · (v) $\tan(3\pi/4)$ · (vi) $\sin(2\pi/3)$ · (vii) $\tan(4\pi/3)$ · (viii) $\cos(3\pi/4)$ · (ix) $\sin(5\pi/6)$ · (x) $\cos(5\pi/3)$.

7D.4 Solve the following equations for $0 \le \theta \le 2\pi$, giving your answers as multiples of π: (i) $\cos\theta = \sqrt{3}/2$ · (ii) $\tan\theta = 1$ · (iii) $\sin\theta = 1/\sqrt{2}$ · (iv) $\sin\theta = -1/2$ · (v) $\cos\theta = -1/\sqrt{2}$ · (vi) $\tan\theta = \sqrt{3}$.

7D.5 Solve the following equations for $-\pi < \theta < \pi$: (i) $\sin\theta = 0.2$ · (ii) $\cos\theta = 0.74$ · (iii) $\tan\theta = 3$ · (iv) $4\sin\theta = -1$ · (v) $\cos\theta = -0.4$ · (vi) $2\tan\theta = -1$.

7D.6 Solve $3\cos^2\theta + 2\sin\theta - 3 = 0$ for $0 \le \theta \le \pi$.

Exercise 7E · Arc length and sector area

7E.1 Each row of the table gives dimensions of a sector of a circle of radius r cm. The angle subtended at the centre of the circle is θ radians, the arc length of the sector is s cm and its area is A cm^2. Copy and complete the table.

r (cm)	θ (rad)	s (cm)	A (cm^2)
5	π/4		
8	1		
4		2	
	π/3	π/2	
5			10
	0.8	1.5	
	2π/3		4π

7E.2 (i) In the diagram, OAB is a sector of a circle of radius 4 cm with centre O and angle AOB = 5π/6 radians; the segment between the chord AB and the arc AB is shaded. (a) Find the area of the sector OAB in the diagram. (b) Show that the area of triangle OAB is 16 sin(5π/12) cos(5π/12). (c) Find the shaded area. (ii) The second diagram shows two circles, each of radius 4 cm, with each one passing through the centre of the other; their overlapping region (with common chord AB) is shaded. Calculate the shaded area. (Hint: Add the common chord AB to the sketch.)

7E.3 The diagram shows the cross-section of three pencils, each of radius 3.5 mm, held together by a stretched elastic band (the three circular cross-sections touch each other mutually, and the small central region enclosed between the three circles is shaded). Find (i) the shaded area (ii) the stretched length of the band.

7E.4 A circle, centre O, has two radii OA and OB. The line AB divides the circle into two regions with areas in the ratio 3 : 1. If the angle AOB is θ (radians), show that $\theta - \sin\theta = \pi/2$.

7E.5 In a cricket match, a particular cricketer generally hits the ball anywhere in a sector of angle 100°. If the boundary (assumed circular) is 80 yards away, find (i) the length of boundary which the fielders should patrol (ii) the area of the ground which the fielders need to cover.

7E.6 In the diagram, ABC is a semi-circle, centre O and radius 9 cm, with diameter AC. The line BD is perpendicular to the diameter AC (with D on OC) and angle AOB = 2.4 radians. The region bounded by OA, OD, DB and arc AB is shaded. (i) Show that BD = 6.08 cm, correct to 3 significant figures. (ii) Find the perimeter of the shaded region. (iii) Find the area of the shaded region. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q8 June 2005]

7E.7 In the diagram, OAB and OCD are radii of a circle, centre O and radius 16 cm. Angle AOC = α radians. AC and BD are arcs of circles, centre O and radii 10 cm and 16 cm respectively, and the region ABDC between the two arcs is shaded. (i) In the case where $\alpha = 0.8$, find the area of the shaded region. (ii) Find the value of α for which the perimeter of the shaded region is 28.9 cm. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q2 November 2005]

7E.8 In the diagram, OAB is a sector of a circle with centre O and radius 12 cm. The lines AX and BX are tangents to the circle at A and B respectively. Angle AOB = $(1/3)\pi$ radians, and the region bounded by AX, BX and arc AB is shaded. (i) Find the exact length of AX, giving your answer in terms of $\sqrt{3}$. (ii) Find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q5 June 2007]

7E.9 In the diagram, the circle has centre O and radius 5 cm. The points P and Q lie on the circle, and the minor arc length PQ is 9 cm. The tangents to the circle at P and Q meet at the point T, and the region bounded by TP, TQ and arc PQ is shaded. Calculate (i) angle POQ in radians (ii) the length of PT (iii) the area of the shaded region. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q6 November 2008]

7E.10 In the diagram, AB is an arc of a circle, centre O and radius r cm, and angle AOB = θ radians. The point X lies on OB and AX is perpendicular to OB; the region AXB bounded by AX, XB and arc AB is shaded. (i) Show that the area, A cm^2, of the shaded region AXB is given by $A = (1/2)r^2(\theta - \sin\theta\cos\theta)$. (ii) In the case where r = 12 and θ = (1/6)π, find the perimeter of the shaded region AXB, leaving your answer in terms of √3 and π. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q7 November 2007]

Exercise 7F · Other trigonometrical functions and transformations

7F.1 Starting with the graph of $y = \sin x$, state the transformations which can be used to sketch each of the following curves: (i) $y = \sin (x - 90°)$ · (ii) $y = \sin 3x$ · (iii) $2y = \sin x$ · (iv) $y = \sin (x/2)$ · (v) $y = 2 + \sin x$.

7F.2 Starting with the graph of $y = \cos x$, state the transformations which can be used to sketch each of the following curves: (i) $y = \cos (x + 60°)$ · (ii) $3y = \cos x$ · (iii) $y = \cos x + 1$ · (iv) $y = \cos 2x$.

7F.3 For each of the following curves (a) sketch the curve (b) identify the curve as being the same as one of the following: $y = \pm\sin x$, $y = \pm\cos x$, or $y = \pm\tan x$. (i) $y = \sin (x + 360°)$ · (ii) $y = \sin (x + 90°)$ · (iii) $y = \tan (x - 180°)$ · (iv) $y = \cos (x - 90°)$ · (v) $y = \cos (x + 180°)$.

7F.4 Starting with the graph of y = tan x, find the equation of the graph and sketch the graph after the following transformations: (i) Translation of (0, 4) (0 units horizontally, +4 units vertically) · (ii) Translation of (−30°, 0) (−30° horizontally, 0 units vertically) · (iii) One-way stretch with scale factor 2 parallel to the x axis.

7F.5 The graph of y = sin x is stretched with scale factor 4 parallel to the y axis. (i) State the equation of the new graph. (ii) Find the exact value of y on the new graph when x = 240°.

7F.6 The function f is defined by f(x) = a + b cos 2x, for $0 \le x \le \pi$. It is given that f(0) = −1 and f((1/2)π) = 7. (i) Find the values of a and b. (ii) Find the x co-ordinates of the points where the curve y = f(x) intersects the x axis. (iii) Sketch the graph of y = f(x). [Cambridge AS & A Level Mathematics 9709, Paper 1 Q8 June 2007]

7F.7 The function f is such that $f(x) = a - b\cos x$ for $0° \leq x \leq 360°$, where a and b are positive constants. The maximum value of f(x) is 10 and the minimum value is −2. (i) Find the values of a and b. (ii) Solve the equation $f(x) = 0$. (iii) Sketch the graph of $y = f(x)$. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q5 November 2008]

7F.8 The diagram shows the graph of $y = a\sin(bx) + c$ for $0 \leq x \leq 2\pi$. (In the diagram, the curve starts at (0, 3), rises to a maximum at y = 9, falls to a minimum at y = −3, returns to y = 3 at $x = \pi$, and completes a second identical cycle between $x = \pi$ and $x = 2\pi$.) (i) Find the values of a, b and c. (ii) Find the smallest value of x in the interval $0 \leq x \leq 2\pi$ for which $y = 0$. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q4 June 2009]

7F.9 The function f is defined by $f : x \mapsto 5 - 3 \sin 2x$ for $0 \leq x \leq \pi$. (i) Find the range of f. (ii) Sketch the graph of $y = f(x)$. (iii) State, with a reason, whether f has an inverse. [Cambridge AS & A Level Mathematics 9709, Paper 12 Q4 November 2009]

7F.10 The function $f : x \mapsto 4 - 3 \sin x$ is defined for the domain $0 \leq x \leq 2\pi$. (i) Solve the equation $f(x) = 2$. (ii) Sketch the graph of $y = f(x)$. (iii) Find the set of values of k for which the equation $f(x) = k$ has no solution. The function $g : x \mapsto 4 - 3 \sin x$ is defined for the domain $(1/2)\pi \leq x \leq A$. (iv) State the largest value of A for which g has an inverse. (v) For this value of A, find the value of $g^{-1}(3)$. [Cambridge AS & A Level Mathematics 9709, Paper 12 Q11 June 2010]

Check yourself

C.1 Given that $\cos\theta = -5/13$ and that $90° < \theta < 180°$, find the exact values of $\sin\theta$ and $\tan\theta$ without using a calculator.

C.2 Prove the identity $(1 + \cos\theta)/(\sin\theta) + (\sin\theta)/(1 + \cos\theta) \equiv 2/(\sin\theta)$.

C.3 Solve the equation $2\cos^2 x + 3\sin x = 3$ for $0° \leq x \leq 360°$.

C.4 Solve the equation $3\sin\theta = 2\cos\theta$ for $-\pi \leq \theta \leq \pi$, giving your answers in radians correct to 3 significant figures.

C.5 A sector OAB of a circle with centre O and radius 8 cm has angle AOB = 1.5 radians. Find (a) the perimeter of the sector OAB, and (b) the area of the segment bounded by the arc AB and the chord AB, correct to 3 significant figures.

C.6 The function f is defined by $f(x) = 3 - 2\cos x$ for $0 \le x \le 2\pi$. (a) State the range of f. (b) Sketch the graph of $y = f(x)$. (c) State the largest value of k for which the restriction of f to $0 \le x \le k$ has an inverse, and find an expression for $f^{-1}(x)$ on this domain.

Chapter quiz

Q1 What is the exact value of $\tan 60° \cos 30°$?

A	B	C	D
3/2	√3/2	√3	1/2

Q2 In which quadrant is sin θ negative and tan θ positive?

A	B	C	D
1st quadrant	2nd quadrant	3rd quadrant	4th quadrant

Q3 sin θ = 3/5 and θ is in the 2nd quadrant. What is tan θ?

A	B	C	D
−3/4	3/4	−4/3	4/3

Q4 Which of these is true for every value of θ?

A	B	C	D
sin θ = cos θ	$\sin^2\theta + \cos^2\theta = 1$	tan θ = 1	sin θ = 1/2

Q5 Solve sin x = −1/2 for 0° ≤ x ≤ 360°.

A	B	C	D
210° and 330°	150° and 210°	30° and 330°	210° and 300°

Q6 Solve tan x = −1 for 0° ≤ x ≤ 360°.

A	B	C	D
135° and 315°	45° and 225°	135° and 225°	315° only

Q7 How many solutions does cos x = 0.4 have for 0° ≤ x ≤ 720°?

A	B	C	D
2	3	4	6

Q8 Convert 5π/6 radians into degrees.

A	B	C	D
150°	120°	135°	300°

Q9 A sector has radius 5 cm and angle 0.8 radians. What is its area?

A	B	C	D
10 cm^2	4 cm^2	20 cm^2	2 cm^2

Q10 A sector has radius 6 cm and angle 2 radians. What is its perimeter?

A	B	C	D
24 cm	12 cm	18 cm	36 cm

Q11 What is the range of $y = 3 + 2 \sin x$?

A	B	C	D
$1 \le y \le 5$	$3 \le y \le 5$	$-1 \le y \le 5$	$2 \le y \le 6$

Q12 Which transformation maps $y = \sin x$ onto $y = \sin (x - 45°)$?

A	B	C	D
Translation 45° to the right	Translation 45° to the left	Stretch, scale factor 45, in x	Reflection in the x axis

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End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/p1-7-trigonometry/