P1·3

Sequences and series

CHAPTER ASSIGNMENT

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. Give exact answers unless a question asks for decimal places. Fractions are written a/b, with brackets where needed.

Practice questions from *Pure Mathematics 1* (Hodder Education, Sophie Goldie), Chapter 3 (Sequences and series), Exercises 3A to 3C, for classroom use. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 3A · Arithmetic progressions

3A.1 Are the following sequences arithmetic? If so, state the common difference and the seventh term. (i) 27, 29, 31, 33, … · (ii) 1, 2, 3, 5, 8, … · (iii) 2, 4, 8, 16, … · (iv) 3, 7, 11, 15, … · (v) 8, 6, 4, 2, …

3A.2 The first term of an arithmetic sequence is −8 and the common difference is 3. (i) Find the seventh term of the sequence. (ii) The last term of the sequence is 100. How many terms are there in the sequence?

3A.3 The first term of an arithmetic sequence is 12, the seventh term is 36 and the last term is 144. (i) Find the common difference. (ii) Find how many terms there are in the sequence.

3A.4 There are 20 terms in an arithmetic progression. The first term is −5 and the last term is 90. (i) Find the common difference. (ii) Find the sum of the terms in the progression.

3A.5 The kth term of an arithmetic progression is given by $u_k = 14 + 2k$. (i) Write down the first three terms of the progression. (ii) Calculate the sum of the first 12 terms of this progression.

3A.6 Below is an arithmetic progression: 120 + 114 + ... + 36. (i) How many terms are there in the progression? (ii) What is the sum of the terms in the progression?

3A.7 The fifth term of an arithmetic progression is 28 and the tenth term is 58. (i) Find the first term and the common difference. (ii) The sum of all the terms in this progression is 444. How many terms are there?

3A.8 The sixth term of an arithmetic progression is twice the third term, and the first term is 3. The sequence has ten terms. (i) Find the common difference. (ii) Find the sum of all the terms in the progression.

3A.9 (i) Find the sum of all the odd numbers between 50 and 150. (ii) Find the sum of all the even numbers from 50 to 150, inclusive. (iii) Find the sum of the terms of the arithmetic sequence with first term 50, common difference 1 and 101 terms. (iv) Explain the relationship between your answers to parts (i), (ii) and (iii).

3A.10 The first term of an arithmetic progression is 3000 and the tenth term is 1200. (i) Find the sum of the first 20 terms of the progression. (ii) After how many terms does the sum of the progression become negative?

3A.11 An arithmetic progression has first term 7 and common difference 3. (i) Write down a formula for the kth term of the progression. Which term of the progression equals 73? (ii) Write down a formula for the sum of the first n terms of the progression. How many terms of the progression are required to give a sum equal to 6300?

3A.12 Paul's starting salary in a company is $14 000 and during the time he stays with the company it increases by $500 each year. (i) What is his salary in his sixth year? (ii) How many years has Paul been working for the company when his total earnings for all his years there are $126 000?

3A.13 A jogger is training for a 10 km charity run. He starts with a run of 400 m; then he increases the distance he runs by 200 m each day. (i) How many days does it take the jogger to reach a distance of 10 km in training? (ii) What total distance will he have run in training by then?

3A.14 A piece of string 10 m long is to be cut into pieces, so that the lengths of the pieces form an arithmetic sequence. (i) The lengths of the longest and shortest pieces are 1 m and 25 cm respectively; how many pieces are there? (ii) If the same string had been cut into 20 pieces with lengths that formed an arithmetic sequence, and if the length of the second longest had been 92.5 cm, how long would the shortest piece have been?

3A.15 The 11th term of an arithmetic progression is 25 and the sum of the first 4 terms is 49. (i) Find the first term of the progression and the common difference. The nth term of the progression is 49. (ii) Find the value of n.

3A.16 The first term of an arithmetic progression is 6 and the fifth term is 12. The progression has n terms and the sum of all the terms is 90. Find the value of n.

3A.17 The training programme of a pilot requires him to fly 'circuits' of an airfield. Each day he flies 3 more circuits than the day before. On the fifth day he flew 14 circuits. Calculate how many circuits he flew: (i) on the first day (ii) in total by the end of the fifth day (iii) in total by the end of the nth day (iv) in total from the end of the nth day to the end of the 2nth day. Simplify your answer.

3A.18 As part of a fund-raising campaign, I have been given some books of raffle tickets to sell. Each book has the same number of tickets and all the tickets I have been given are numbered in sequence. The number of the ticket on the front of the 5th book is 205 and that on the front of the 19th book is 373. (i) By writing the number of the ticket on the front of the first book as a and the number of tickets in each book as d, write down two equations involving a and d. (ii) From these two equations find how many tickets are in each book and the number on the front of the first book I have been given. (iii) The last ticket I have been given is numbered 492. How many books have I been given?

Exercise 3B · Geometric progressions

3B.1 Are the following sequences geometric? If so, state the common ratio and calculate the seventh term. (i) 5, 10, 20, 40, … · (ii) 2, 4, 6, 8, … · (iii) 1, −1, 1, −1, … · (iv) 5, 5, 5, 5, … · (v) 6, 3, 0, −3, … · (vi) 6, 3, 1 1/2, 3/4, … · (vii) 1, 1.1, 1.11, 1.111, …

3B.2 A geometric sequence has first term 3 and common ratio 2. The sequence has eight terms. (i) Find the last term. (ii) Find the sum of the terms in the sequence.

3B.3 The first term of a geometric sequence of positive terms is 5 and the fifth term is 1280. (i) Find the common ratio of the sequence. (ii) Find the eighth term of the sequence.

3B.4 A geometric sequence has first term 1/9 and common ratio 3. (i) Find the fifth term. (ii) Which is the first term of the sequence which exceeds 1000?

3B.5 (i) Find how many terms there are in the geometric sequence 8, 16, ..., 2048. (ii) Find the sum of the terms in this sequence.

3B.6 (i) Find how many terms there are in the geometric sequence 200, 50, ..., 0.195 312 5. (ii) Find the sum of the terms in this sequence.

3B.7 The fifth term of a geometric progression is 48 and the ninth term is 768. All the terms are positive. (i) Find the common ratio. (ii) Find the first term. (iii) Find the sum of the first ten terms.

3B.8 The first three terms of an infinite geometric progression are 4, 2 and 1. (i) State the common ratio of this progression. (ii) Calculate the sum to infinity of its terms.

3B.9 The first three terms of an infinite geometric progression are 0.7, 0.07, 0.007. (i) Write down the common ratio for this progression. (ii) Find, as a fraction, the sum to infinity of the terms of this progression. (iii) Find the sum to infinity of the geometric progression 0.7 − 0.07 + 0.007 − ..., and hence show that 7/11 = 0.6363... (the digits 6 and 3 recurring).

3B.10 The first three terms of a geometric sequence are 100, 90 and 81. (i) Write down the common ratio of the sequence. (ii) Which is the position of the first term in the sequence that has a value less than 1? (iii) Find the sum to infinity of the terms of this sequence. (iv) After how many terms is the sum of the sequence greater than 99% of the sum to infinity?

3B.11 A geometric progression has first term 4 and its sum to infinity is 5. (i) Find the common ratio. (ii) Find the sum to infinity if the first term is excluded from the progression.

3B.12 (i) The third term of a geometric progression is 16 and the fourth term is 12.8. Find the common ratio and the first term. (ii) The sum of the first n terms of a geometric progression is $2(2^n + 1) - 2$. Find the first term and the common ratio.

3B.13 (i) The first two terms of a geometric series are 3 and 4. Find the third term. (ii) Given that x, 4, x + 6 are consecutive terms of a geometric series, find: (a) the possible values of x (b) the corresponding values of the common ratio of the geometric series. (iii) Given that x, 4, x + 6 are the sixth, seventh and eighth terms of a geometric series and that the sum to infinity of the series exists, find: (a) the first term (b) the sum to infinity.

3B.14 The first four terms in an infinite geometric series are 54, 18, 6, 2. (i) What is the common ratio r? (ii) Write down an expression for the nth term of the series. (iii) Find the sum of the first n terms of the series. (iv) Find the sum to infinity. (v) How many terms are needed for the sum to be greater than 80.999?

3B.15 A tank is filled with 20 litres of water. Half the water is removed and replaced with anti-freeze and thoroughly mixed. Half this mixture is then removed and replaced with anti-freeze. The process continues. (i) Find the first five terms in the sequence of amounts of water in the tank at each stage. (ii) Find the first five terms in the sequence of amounts of anti-freeze in the tank at each stage. (iii) Is either of these sequences geometric? Explain.

3B.16 A pendulum is set swinging. Its first oscillation is through an angle of 30°, and each succeeding oscillation is through 95% of the angle of the one before it. (i) After how many swings is the angle through which it swings less than 1°? (ii) What is the total angle it has swung through at the end of its tenth oscillation?

3B.17 A ball is thrown vertically upwards from the ground. It rises to a height of 10 m and then falls and bounces. After each bounce it rises vertically to 2/3 of the height from which it fell. (i) Find the height to which the ball bounces after the nth impact with the ground. (ii) Find the total distance travelled by the ball from the first throw to the tenth impact with the ground.

3B.18 The first three terms of an arithmetic sequence, a, a + d and a + 2d, are the same as the first three terms, a, ar, ar^2, of a geometric sequence ($a \neq 0$). Show that this is only possible if r = 1 and d = 0.

3B.19 The first term of a geometric progression is 81 and the fourth term is 24. Find (i) the common ratio of the progression (ii) the sum to infinity of the progression. The second and third terms of this geometric progression are the first and fourth terms respectively of an arithmetic progression. (iii) Find the sum of the first ten terms of the arithmetic progression.

3B.20 A progression has a second term of 96 and a fourth term of 54. Find the first term of the progression in each of the following cases: (i) the progression is arithmetic (ii) the progression is geometric with a positive common ratio.

3B.21 (i) Find the sum to infinity of the geometric progression with first three terms 0.5, 0.5^3 and 0.5^5. (ii) The first two terms in an arithmetic progression are 5 and 9. The last term in the progression is the only term which is greater than 200. Find the sum of all the terms in the progression.

3B.22 The 1st term of an arithmetic progression is a and the common difference is d, where $d \neq 0$. (i) Write down expressions, in terms of a and d, for the 5th term and the 15th term. The 1st term, the 5th term and the 15th term of the arithmetic progression are the first three terms of a geometric progression. (ii) Show that $3a = 8d$. (iii) Find the common ratio of the geometric progression.

Exercise 3C · Binomial expansions

3C.1 Write out the following binomial expansions. (i) $(x + 1)^4$ · (ii) $(1 + x)^7$ · (iii) $(x + 2)^5$ · (iv) $(2x + 1)^6$ · (v) $(2x - 3)^4$ · (vi) $(2x + 3y)^3$ · (vii) $(x - 2/x)^3$ · (viii) $(x + 2/x^2)^4$ · (ix) $(3x^2 - 2/x)^5$

3C.2 Use a non-calculator method to calculate the following binomial coefficients. Check your answers using your calculator's shortest method. (i) C(4, 2) · (ii) C(6, 2) · (iii) C(6, 3) · (iv) C(6, 4) · (v) C(6, 0) · (vi) C(12, 9) (here C(n, r) is the coefficient written ${}^nC_r$, or n over r in a bracket)

3C.3 In these expansions, find the coefficient of these terms. (i) x^5 in $(1 + x)^8$ · (ii) x^4 in $(1 – x)^{10}$ · (iii) x^6 in $(1 + 3x)^{12}$ · (iv) x^7 in $(1 – 2x)^{15}$ · (v) x^2 in $(x^2 + 2/x)^{10}$

3C.4 (i) Simplify $(1 + x)^3 – (1 – x)^3$. (ii) Show that $a^3 – b^3 = (a – b)(a^2 + ab + b^2)$. (iii) Substitute $a = 1 + x$ and $b = 1 – x$ in the result in part (ii) and show that your answer is the same as that for part (i).

3C.5 Find the first three terms, in descending powers of x, in the expansion of $(2x – 2/x)^4$.

3C.6 Find the first three terms, in ascending powers of x, in the expansion $(2 + kx)^6$.

3C.7 (i) Find the first three terms, in ascending powers of x, in the expansion $(1 - 2x)^6$. (ii) Hence find the coefficients of x and x^2 in the expansion of $(4 - x)(2 - 4x)^6$.

3C.8 (i) Find the first three terms, in descending powers of x, in the expansion $(4x - k/x^2)^6$. (ii) Given that the value of the term in the expansion which is independent of x is 240, find possible values of k.

3C.9 (i) Find the first three terms, in descending powers of x, in the expansion of $(x^2 - 1/x)^6$. (ii) Find the coefficient of x^3 in the expansion of $(x^2 - 1/x)^6$.

3C.10 (i) Find the first three terms, in descending powers of x, in the expansion of $(x - 2/x)^5$. (ii) Hence find the coefficient of x in the expansion of $(4 + 1/x^2)(x - 2/x)^5$.

3C.11 (i) Show that $(2 + x)^4 = 16 + 32x + 24x^2 + 8x^3 + x^4$ for all x. (ii) Find the values of x for which $(2 + x)^4 = 16 + 16x + x^4$.

3C.12 The first three terms in the expansion of $(2 + ax)^n$, in ascending powers of x, are $32 - 40x + bx^2$. Find the values of the constants n, a and b.

3C.13 (i) Find the first three terms in the expansion of $(2 - x)^6$ in ascending powers of x. (ii) Find the value of k for which there is no term in x^2 in the expansion of $(1 + kx)(2 - x)^6$.

3C.14 (i) Find the first three terms in the expansion of $(1 + ax)^5$ in ascending powers of x. (ii) Given that there is no term in x in the expansion of $(1 - 2x)(1 + ax)^5$, find the value of the constant a. (iii) For this value of a, find the coefficient of x^2 in the expansion of $(1 - 2x)(1 + ax)^5$.

Check yourself

C1 An arithmetic progression has first term 4 and common difference 5. (a) Find the 30th term. (b) Find the sum of the first 30 terms.

C2 A geometric progression has first term 6 and common ratio 1/3. (a) Find the 5th term. (b) Find the sum to infinity.

C3 The 3rd term of an arithmetic progression is 19 and the 8th term is 44. Find the first term, the common difference and the sum of the first 12 terms.

C4 The first three terms of a geometric progression are 5, 15 and 45. Find the least number of terms needed for the sum to exceed 10^6.

C5 Find the coefficient of x^3 in the expansion of $(2 - x)^7$.

C6 Expand $(1 + 2x)^5$ fully, and use your answer to estimate 1.02^5 to 4 decimal places (put $x = 0.01$ in the first four terms).

Chapter quiz

Q1 What is Σ (k = 1 to 4) of (2k + 1)?

A	B	C	D
16	24	20	9

Q2 An AP has $u_k = 3k - 2$. What is the 10th term?

A	B	C	D
32	25	28	30

Q3 Find the 20th term of the AP 7, 11, 15, ...

A	B	C	D
87	83	79	80

Q4 How many terms are in the AP 5, 9, 13, ..., 101?

A	B	C	D
24	26	25	23

Q5 Find the sum of the first 10 terms of the AP 2, 5, 8, ...

A	B	C	D
145	155	150	310

Q6 Find the 8th term of the GP 2, 6, 18, ...

A	B	C	D
4374	13 122	1458	2187

Q7 Find the sum of the first 5 terms of the GP 4, 8, 16, ...

A	B	C	D
64	128	124	120

Q8 Find the sum to infinity of the GP 18, 6, 2, ...

A	B	C	D
24	27	36	54

Q9 When does $a + ar + ar^2 + \ldots$ have a sum to infinity?

A	B	C	D
$r < 1$	$r > 1$	$-1 < r < 1$	$0 < r < 2$

Q10 What is the coefficient of x^2 in the expansion of $(1 + x)^6$?

A	B	C	D
6	20	30	15

Q11 What is the coefficient of x^3 in the expansion of $(2 + x)^5$?

A	B	C	D
40	80	10	20

Q12 What is the value of ${}^7C_2$?

A	B	C	D
14	42	35	21

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/p1-3-sequences-and-series/