P1·2

Coordinate geometry

CHAPTER ASSIGNMENT

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. Give exact answers (surds and fractions) unless a question asks for decimal places. Fractions are written a/b, with brackets where needed.

Practice questions from *Pure Mathematics 1* (Hodder Education, Sophie Goldie), Chapter 2 (Co-ordinate geometry), Exercises 2A to 2F, for classroom use. Where a question needs a diagram, the diagram is described in words. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 2A · Gradient, mid-point and distance between two points

2A.1 For the following pairs of points A and B, calculate: (a) the gradient of the line AB, (b) the mid-point of the line joining A to B, (c) the distance AB, (d) the gradient of the line perpendicular to AB. (i) A(0, 1), B(2, −3) · (ii) A(3, 2), B(4, −1) · (iii) A(−6, 3), B(6, 3) · (iv) A(5, 2), B(2, −8) · (v) A(4, 3), B(2, 0) · (vi) A(1, 4), B(1, −2)

2A.2 The line joining the point P(3, −4) to Q(q, 0) has a gradient of 2. Find the value of q.

2A.3 The three points X(2, −1), Y(8, y) and Z(11, 2) are collinear (i.e. they lie on the same straight line). Find the value of y.

2A.4 The points A, B, C and D have co-ordinates (1, 2), (7, 5), (9, 8) and (3, 5). (i) Find the gradients of the lines AB, BC, CD and DA. (ii) What do these gradients tell you about the quadrilateral ABCD? (iii) Draw a diagram to check your answer to part (ii).

2A.5 The points A, B and C have co-ordinates (2, 1), (b, 3) and (5, 5), where b > 3 and ∠ABC = 90°. Find: (i) the value of b, (ii) the lengths of AB and BC, (iii) the area of triangle ABC.

2A.6 The triangle PQR has vertices P(8, 6), Q(0, 2) and R(2, r). Find the values of r when the triangle: (i) has a right angle at P, (ii) has a right angle at Q, (iii) has a right angle at R, (iv) is isosceles with RQ = RP.

2A.7 The points A, B, and C have co-ordinates (−4, 2), (7, 4) and (−3, −1). (i) Draw the triangle ABC. (ii) Show by calculation that the triangle ABC is isosceles and name the two equal sides. (iii) Find the mid-point of the third side. (iv) By calculating appropriate lengths, calculate the area of the triangle ABC.

2A.8 For the points P(x, y), and Q(3x, 5y), find in terms of x and y: (i) the gradient of the line PQ, (ii) the mid-point of the line PQ, (iii) the length of the line PQ.

2A.9 A quadrilateral has vertices A(0, 0), B(0, 3), C(6, 6) and D(12, 6). (i) Draw the quadrilateral. (ii) Show by calculation that it is a trapezium. (iii) Find the co-ordinates of E when EBCD is a parallelogram.

2A.10 Three points A, B and C have co-ordinates (1, 3), (3, 5) and (−1, y). Find the values of y when: (i) AB = AC, (ii) AC = BC, (iii) AB is perpendicular to BC, (iv) A, B and C are collinear.

2A.11 The diagonals of a rhombus bisect each other at 90°, and conversely, when two lines bisect each other at 90°, the quadrilateral formed by joining the end points of the lines is a rhombus. Use the converse result to show that the points with co-ordinates (1, 2), (8, −2), (7, 6) and (0, 10) are the vertices of a rhombus, and find its area.

Exercise 2B · Sketching lines and parallel or perpendicular lines

2B.1 Sketch the following lines: (i) $y = -2$ · (ii) $x = 5$ · (iii) $y = 2x$ · (iv) $y = -3x$ · (v) $y = 3x + 5$ · (vi) $y = x - 4$ · (vii) $y = x + 4$ · (viii) $y = (1/2)x + 2$ · (ix) $y = 2x + 1/2$ · (x) $y = -4x + 8$ · (xi) $y = 4x - 8$ · (xii) $y = -x + 1$ · (xiii) $y = -(1/2)x - 2$ · (xiv) $y = 1 - 2x$ · (xv) $3x - 2y = 6$ · (xvi) $2x + 5y = 10$ · (xvii) $2x + y - 3 = 0$ · (xviii) $2y = 5x - 4$ · (xix) $x + 3y - 6 = 0$ · (xx) $y = 2 - x$

2B.2 By calculating the gradients of the following pairs of lines, state whether they are parallel, perpendicular or neither: (i) $y = -4$ and $x = 2$ · (ii) $y = 3x$ and $x = 3y$ · (iii) $2x + y = 1$ and $x - 2y = 1$ · (iv) $y = 2x + 3$ and $4x - y + 1 = 0$ · (v) $3x - y + 2 = 0$ and $3x + y = 0$ · (vi) $2x + 3y = 4$ and $2y = 3x - 2$ · (vii) $x + 2y - 1 = 0$ and $x + 2y + 1 = 0$ · (viii) $y = 2x - 1$ and $2x - y + 3 = 0$ · (ix) $y = x - 2$ and $x + y = 6$ · (x) $y = 4 - 2x$ and $x + 2y = 8$ · (xi) $x + 3y - 2 = 0$ and $y = 3x + 2$ · (xii) $y = 2x$ and $4x + 2y = 5$

Exercise 2C · Finding the equation of a line

2C.1 Find the equations of the lines (i) – (x) in the diagrams below. (The diagrams are described here.) (i) A vertical line passing through (7, 0). (ii) A horizontal line passing through (0, 5). (iii) A line passing through (0, 0) and (2, 4). (iv) A line passing through (0, 2) and (2, 0). (v) A line passing through (−4, −2), (0, −3) and (4, −4). (vi) A line passing through (0, 0) and (4, 4). (vii) A vertical line passing through (−4, 0). (viii) A horizontal line passing through (0, −4). (ix) A line passing through (0, 0) and (4, −2). (x) A line passing through (0, 4), (3, 3) and (6, 2).

2C.2 Find the equations of the following lines: (i) parallel to $y = 2x$ and passing through $(1, 5)$ · (ii) parallel to $y = 3x - 1$ and passing through $(0, 0)$ · (iii) parallel to $2x + y - 3 = 0$ and passing through $(-4, 5)$ · (iv) parallel to $3x - y - 1 = 0$ and passing through $(4, -2)$ · (v) parallel to $2x + 3y = 4$ and passing through $(2, 2)$ · (vi) parallel to $2x - y - 8 = 0$ and passing through $(-1, -5)$

2C.3 Find the equations of the following lines: (i) perpendicular to $y = 3x$ and passing through $(0, 0)$ · (ii) perpendicular to $y = 2x + 3$ and passing through $(2, -1)$ · (iii) perpendicular to $2x + y = 4$ and passing through $(3, 1)$ · (iv) perpendicular to $2y = x + 5$ and passing through $(-1, 4)$ · (v) perpendicular to $2x + 3y = 4$ and passing through $(5, -1)$ · (vi) perpendicular to $4x - y + 1 = 0$ and passing through $(0, 6)$

2C.4 Find the equations of the line AB in each of the following cases: (i) A(0, 0), B(4, 3) · (ii) A(2, −1), B(3, 0) · (iii) A(2, 7), B(2, −3) · (iv) A(3, 5), B(5, −1) · (v) A(−2, 4), B(5, 3) · (vi) A(−4, −2), B(3, −2)

2C.5 Triangle ABC has an angle of 90° at B. Point A is on the y axis, AB is part of the line x − 2y + 8 = 0 and C is the point (6, 2). (i) Sketch the triangle. (ii) Find the equations of AC and BC. (iii) Find the lengths of AB and BC and hence find the area of the triangle. (iv) Using your answer to part (iii), find the length of the perpendicular from B to AC.

2C.6 A median of a triangle is a line joining one of the vertices to the mid-point of the opposite side. In a triangle OAB, O is at the origin, A is the point (0, 6) and B is the point (6, 0). (i) Sketch the triangle. (ii) Find the equations of the three medians of the triangle. (iii) Show that the point (2, 2) lies on all three medians. (This shows that the medians of this triangle are concurrent.)

2C.7 A quadrilateral ABCD has its vertices at the points (0, 0), (12, 5), (0, 10) and (−6, 8) respectively. (i) Sketch the quadrilateral. (ii) Find the gradient of each side. (iii) Find the length of each side. (iv) Find the equation of each side. (v) Find the area of the quadrilateral.

Exercise 2D · The intersection of two lines

2D.1 (i) Find the vertices of the triangle ABC whose sides are given by the lines AB: $x - 2y = -1$, BC: $7x + 6y = 53$ and AC: $9x + 2y = 11$. (ii) Show that the triangle is isosceles.

2D.2 Two sides of a parallelogram are formed by parts of the lines $2x - y = -9$ and $x - 2y = -9$. (i) Show these two lines on a graph. (ii) Find the co-ordinates of the vertex where they intersect. Another vertex of the parallelogram is the point (2, 1). (iii) Find the equations of the other two sides of the parallelogram. (iv) Find the co-ordinates of the other two vertices.

2D.3 A(0, 1), B(1, 4), C(4, 3) and D(3, 0) are the vertices of a quadrilateral ABCD. (i) Find the equations of the diagonals AC and BD. (ii) Show that the diagonals AC and BD bisect each other at right angles. (iii) Find the lengths of AC and BD. (iv) What type of quadrilateral is ABCD?

2D.4 The line with equation $5x + y = 20$ meets the x axis at A and the line with equation $x + 2y = 22$ meets the y axis at B. The two lines intersect at a point C. (i) Sketch the two lines on the same diagram. (ii) Calculate the co-ordinates of A, B and C. (iii) Calculate the area of triangle OBC where O is the origin. (iv) Find the co-ordinates of the point E such that ABEC is a parallelogram.

2D.5 A median of a triangle is a line joining a vertex to the mid-point of the opposite side. In any triangle, the three medians meet at a point. The centroid of a triangle is at the point of intersection of the medians. Find the co-ordinates of the centroid for each triangle shown. (The diagrams are described here.) (i) A right-angled triangle with vertices O(0, 0), (6, 0) and (0, 12). (ii) An isosceles triangle with vertices (−5, 0), (5, 0) and (0, 9).

2D.6 You are given the co-ordinates of the four points A(6, 2), B(2, 4), C(−6, −2) and D(−2, −4). (i) Calculate the gradients of the lines AB, CB, DC and DA. Hence describe the shape of the figure ABCD. (ii) Show that the equation of the line DA is $4y - 3x = -10$ and find the length DA. (iii) Calculate the gradient of a line which is perpendicular to DA and hence find the equation of the line l through B which is perpendicular to DA. (iv) Calculate the co-ordinates of the point P where l meets DA. (v) Calculate the area of the figure ABCD. [MEI]

2D.7 The diagram shows a triangle whose vertices are A(−2, 1), B(1, 7) and C(3, 1). The point L is the foot of the perpendicular from A to BC, and M is the foot of the perpendicular from B to AC. (i) Find the gradient of the line BC. (ii) Find the equation of the line AL. (iii) Write down the equation of the line BM. The lines AL and BM meet at H. (iv) Find the co-ordinates of H. (v) Show that CH is perpendicular to AB. (vi) Find the area of the triangle BLH. [MEI]

2D.8 The diagram shows a rectangle ABCD. The point A is (0, −2) and C is (12, 14). The diagonal BD is parallel to the x axis (with B to the left of AC and D to the right of AC). (i) Explain why the y co-ordinate of D is 6. The x co-ordinate of D is h. (ii) Express the gradients of AD and CD in terms of h. (iii) Calculate the x co-ordinates of D and B. (iv) Calculate the area of the rectangle ABCD. [Cambridge AS & A Level Mathematics 9709, Paper 12 Q9 November 2009]

2D.9 The diagram shows a rhombus ABCD. The points B and D have co-ordinates (2, 10) and (6, 2) respectively, and A lies on the x axis. The mid-point of BD is M. Find, by calculation, the co-ordinates of each of M, A and C. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q5 June 2005]

2D.10 Three points have co-ordinates A(2, 6), B(8, 10) and C(6, 0). The perpendicular bisector of AB meets the line BC at D. Find (i) the equation of the perpendicular bisector of AB in the form ax + by = c, (ii) the co-ordinates of D. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q7 November 2005]

2D.11 The diagram shows a rectangle ABCD. The point A is (2, 14), B is (−2, 8) and C lies on the x axis. Find (i) the equation of BC, (ii) the co-ordinates of C and D. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q6 June 2007]

2D.12 The three points A(3, 8), B(6, 2) and C(10, 2) are shown in the diagram. The point D is such that the line DA is perpendicular to AB and DC is parallel to AB. Calculate the co-ordinates of D. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q6 November 2007]

2D.13 In the diagram, the points A and C lie on the x and y axes respectively and the equation of AC is $2y + x = 16$. The point B has co-ordinates (2, 2). The perpendicular from B to AC meets AC at the point X. (i) Find the co-ordinates of X. The point D is such that the quadrilateral ABCD has AC as a line of symmetry. (ii) Find the co-ordinates of D. (iii) Find, correct to 1 decimal place, the perimeter of ABCD. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q11 June 2008]

2D.14 The diagram shows points A, B and C lying on the line $2y = x + 4$ (in that order along the line). The point A lies on the y axis and AB = BC. The line from D(10, −3) to B is perpendicular to AC. Calculate the co-ordinates of B and C. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q8 June 2009]

Exercise 2E · Sketching polynomial curves

2E.1 Sketch the curve $y = x(x - 3)(x + 4)$, marking clearly the values of x and y where it crosses the co-ordinate axes.

2E.2 Sketch the curve $y = (x + 1)(2x - 5)(x - 4)$, marking clearly the values of x and y where it crosses the co-ordinate axes.

2E.3 Sketch the curve $y = (5 - x)(x - 1)(x + 3)$, marking clearly the values of x and y where it crosses the co-ordinate axes.

2E.4 Sketch the curve $y = x^2(x - 3)$, marking clearly the values of x and y where it crosses the co-ordinate axes.

2E.5 Sketch the curve $y = (x + 1)^2(2 - x)$, marking clearly the values of x and y where it crosses the co-ordinate axes.

2E.6 Sketch the curve $y = (3x - 4)(4x - 3)^2$, marking clearly the values of x and y where it crosses the co-ordinate axes.

2E.7 Sketch the curve $y = (x + 2)^2(x - 4)^2$, marking clearly the values of x and y where it crosses the co-ordinate axes.

2E.8 Sketch the curve $y = (x - 3)^2(4 + x)^2$, marking clearly the values of x and y where it crosses the co-ordinate axes.

2E.9 Suggest an equation for this curve. (In the diagram, a W-shaped quartic curve opens upwards; it touches the x axis at (−1, 0) and at (2, 0), and crosses the y axis at (0, 4).)

Exercise 2F · The intersection of a line and a curve

2F.1 Show that the line $y = 3x + 1$ crosses the curve $y = x^2 + 3$ at (1, 4) and find the co-ordinates of the other point of intersection.

2F.2 (i) Find the co-ordinates of the points A and B where the line $y = 2x - 1$ cuts the curve $y = x^2 - 4$. (ii) Find the distance AB.

2F.3 (i) Find the co-ordinates of the points of intersection of the line $y = 2x$ and the curve $y = x^2 + 6x - 5$. (ii) Show also that the line $y = 2x$ does not cross the curve $y = x^2 + 6x + 5$.

2F.4 The line $3y = 5 - x$ intersects the curve $2y^2 = x$ at two points. Find the distance between the two points.

2F.5 The equation of a curve is $xy = 8$ and the equation of a line is $2x + y = k$, where k is a constant. Find the values of k for which the line forms a tangent to the curve.

2F.6 Find the value of the constant c for which the line $y = 4x + c$ is a tangent to the curve $y^2 = 4x$.

2F.7 The equation of a curve is $xy = 10$ and the equation of a line l is $2x + y = q$, where q is a number. (i) In the case where $q = 9$, find the co-ordinates of the points of intersection of l and the curve. (ii) Find the set of values of q for which l does not intersect the curve.

2F.8 The curve $y^2 = 12x$ intersects the line $3y = 4x + 6$ at two points. Find the distance between the two points. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q5 June 2006]

2F.9 Determine the set of values of the constant k for which the line $y = 4x + k$ does not intersect the curve $y = x^2$. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q1 November 2007]

2F.10 Find the set of values of k for which the line $y = kx - 4$ intersects the curve $y = x^2 - 2x$ at two distinct points. [Cambridge AS & A Level Mathematics 9709, Paper 1 Q2 June 2009]

Check yourself

C.1 Find the equation of the perpendicular bisector of the line joining A(−2, 7) and B(6, 3), giving your answer in the form $ax + by + c = 0$.

C.2 The points P(1, −2), Q(5, 4) and R(k, 1) form a triangle with a right angle at Q. Find the value of k and the exact area of triangle PQR.

C.3 Find the co-ordinates of the point of intersection of the lines $3x - 2y = 13$ and $2x + 5y = -4$.

C.4 Sketch the curve $y = (x + 2)^2(x - 3)$, showing clearly the co-ordinates of the points where the curve meets the co-ordinate axes.

C.5 Write the circle $x^2 + y^2 - 6x + 8y - 11 = 0$ in the form $(x - a)^2 + (y - b)^2 = r^2$, and state its centre and radius.

C.6 Find the values of the constant k for which the line $y = x + k$ is a tangent to the circle $x^2 + y^2 = 18$, and find the co-ordinates of the point of contact in each case.

Chapter quiz

Q1 What is the gradient of the line through (2, −1) and (6, 7)?

A	B	C	D
½	−2	2	8/3

Q2 What is the distance between (−1, 2) and (4, 14)?

A	B	C	D
13	17	√109	7

Q3 What is the mid-point of the line joining (−3, 8) and (7, −2)?

A	B	C	D
(4, 5)	(5, 5)	(2, 5)	(2, 3)

Q4 A line is perpendicular to $3x + 2y = 6$. What is its gradient?

A	B	C	D
−2/3	2/3	3/2	−3/2

Q5 Which is the line through (1, 5) with gradient −2?

A	B	C	D
y = −2x + 3	y = 2x + 3	y = −2x + 5	y = −2x + 7

Q6 Where do the lines $x + y = 7$ and $2x - y = 2$ meet?

A	B	C	D
(4, 3)	(2, 5)	(3, 4)	(5, 2)

Q7 How does the curve $y = (x - 2)^2(x + 1)$ meet the x axis?

A	B	C	D
Crosses at $x = 2$, touches at $x = -1$	Touches at $x = 2$, crosses at $x = -1$	Touches the axis at both $x = 2$ and $x = -1$	Crosses the axis at both $x = 2$ and $x = -1$

Q8 What happens to $y = -3x^4 + 2x$ as $x \to \infty$ and as $x \to -\infty$?

A	B	C	D
$y \to -\infty$ at both ends	$y \to +\infty$ at both ends	$y \to -\infty$ as $x \to \infty$ and $y \to +\infty$ as $x \to -\infty$	$y \to +\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

Q9 What are the centre and radius of $(x + 2)^2 + (y - 3)^2 = 49$?

A	B	C	D
Centre (2, −3), radius 7	Centre (−2, 3), radius 49	Centre (−2, 3), radius 7	Centre (2, −3), radius 49

Q10 What are the centre and radius of $x^2 + y^2 - 10x + 4y + 13 = 0$?

A	B	C	D
Centre (−5, 2), radius 4	Centre (5, −2), radius 4	Centre (5, −2), radius 16	Centre (10, −4), radius 4

Q11 The line $y = 2x + k$ is a tangent to $y = x^2 + 3$. What is k?

A	B	C	D
3	−2	1	2

Q12 A circle has centre O and passes through (3, 4). What is the gradient of the tangent at (3, 4)?

A	B	C	D
−3/4	3/4	−4/3	4/3
