M1·5

Energy, work and power

CHAPTER ASSIGNMENT

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. Take g = 10 m s^{-2} unless a question specifies otherwise. State appropriate units and degrees of accuracy. Give non-exact values correct to 3 significant figures unless stated otherwise.

Practice questions from *Cambridge International AS & A Level Mathematics: Mechanics Student's Book* (Collins, Tom Andrews and Michael Kent), Chapter 5 (Energy, work and power), Exercises 5.1A to 5.4A, and Exam-style questions, for classroom use. Where a question needs a diagram, data or descriptions are provided. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 5.1A · Work done by a constant force

5.1A.1 Find the work done by:

(a) a horizontal force of 4 N pushing a lamp 2.1 m across a table

(b) a horizontal force of 5.2 kN pushing a car 1.3 m across a road

(c) a horizontal force of 9.3 N pushing a lawnmower 80 cm across a lawn.

5.1A.2 Find the work done by:

(a) a 6 N force acting at 18° pushing a bag 4 m across a floor

(b) a 17 N force acting at an angle of $\cos^{-1}(4/5)$ pushing a hot water bottle 40 cm across a table

(c) a 380 kN force acting at an angle of $\tan^{-1}(12/35)$ pushing a truck 7.4 m across a car park.

5.1A.3

(a) An armchair is pushed 2.5 m across a floor. Given that the work done by the force is 400 J, find the force applied to the armchair.

(b) A sofa is pushed by a force of 750 N across a floor. Given that the work done by the force is 1.05 kJ, find the distance moved by the sofa.

5.1A.4 A crate of bricks is lifted through a vertical distance of 18 m. The crate of bricks has a mass of 900 kg. Find the work done against gravity.

5.1A.5 A refrigerator of mass 130 kg is pulled across a rough kitchen floor. The magnitude of the frictional force is 440 N. Given that the work done against friction is 528 J, find the distance the refrigerator is pulled.

5.1A.6 A table is raised vertically 5.4 m to the second floor of a house. Given that the work done against gravity is 2295 J, find the mass of the table.

5.1A.7 A van of weight 16 kN is lowered through a vertical distance of 6.5 m. Find the work done by gravity in lowering the van.

Exercise 5.1B · Work done on slopes and against friction

5.1B.1 A trolley of mass 16 kg is pushed at a constant speed of 2.5 m s^{-1} for 8 s across a rough horizontal floor.

(a) Find the distance travelled by the trolley.

The force pushing the trolley, X N, is applied at an angle of 6° below the horizontal.

(b) Find an expression for the friction force in terms of X.

(c) Find an expression for the normal reaction force in terms of X.

The coefficient of friction between the trolley and the floor is 0.3.

(d) Find the value of X.

(e) Find the work done by the force.

5.1B.2 A shed is pushed 12.2 m up a rough plane inclined at $\cos^{-1}(60/61)$ to the horizontal by a force acting parallel to the plane. The mass of the shed is 244 kg. The coefficient of friction between the shed and the plane is 2/15.

(a) Find the magnitude of the force exerted by the plane on the shed (the normal reaction).

(b) Find the magnitude of the frictional force.

(c) Find the work done against friction.

(d) Find the vertical distance moved by the shed.

(e) Find the work done against gravity.

(f) Find the total work done moving the shed, in kJ correct to 3 significant figures.

5.1B.3 A plane is inclined at an angle of 8.3° to the horizontal. A block of mass 9.5 kg is pulled 3.8 m up the plane. The plane is modelled as a rough surface and the block is modelled as a particle. The coefficient of friction between the block and plane is 0.22.

(a) Find the work done against friction.

(b) Find the work done against gravity.

5.1B.4 A wheelbarrow of mass 25 kg is pushed up a rough track inclined at an angle of 30° to the horizontal at a speed of 0.2 m s^{-1}. Given that the coefficient of friction between the wheelbarrow and the track is $\sqrt{3} / 15$, find the total work done in one minute.

5.1B.5 A particle of mass 3.6 kg is pulled 4.2 m at a constant speed up a rough inclined plane. The total work done is 43 J and the plane is inclined at an angle of 10° to the horizontal. Find the coefficient of friction between the particle and the plane.

5.1B.6 A box of mass 6 kg is pushed up a rough driveway inclined at an angle of $\tan^{-1}(3/4)$ to the horizontal. The coefficient of friction between the box and the driveway is 3/8. Given that the total work done is 513 J, find the distance the box is pushed up the driveway.

5.1B.7 A hill is inclined at an angle of α to the horizontal. The hill is modelled as a rough plane with a coefficient of friction of 0.24. A young boy on a toboggan is pushed 15 m up the hill. The total mass of the boy and the toboggan is 51 kg. Given that the work done against gravity is 3.6 kJ, find:

(a) the value of $\sin \alpha$

(b) the work done against friction.

Exercise 5.2A · Kinetic and potential energy

5.2A.1

(a) A particle has a mass of 8 kg and is travelling at 1.5 m s^{-1}. Calculate its kinetic energy.

(b) Find the kinetic energy of a car of mass 1200 kg moving at 10 m s^{-1}. Give the answer in kilojoules (kJ).

(c) A particle of mass 450 g has an initial velocity of 3 m s^{-1} and a final velocity of 9 m s^{-1}. Find the change in kinetic energy, stating whether it is an increase or a decrease.

(d) Find the change in kinetic energy when a parcel of mass 15 kg reduces in speed from 6.3 m s^{-1} to 2.7 m s^{-1}, stating whether it is an increase or a decrease.

5.2A.2

(a) A box of mass 4 kg is lifted 2.3 m vertically. Find the change in potential energy, stating whether it is an increase or a decrease.

(b) A particle is raised vertically through 12 m. Given that the particle has a mass of 0.9 kg, find the increase in the potential energy of the particle.

(c) A man of mass 80 kg climbs 4 m down a ladder. Find the decrease in the potential energy of the man.

(d) A lift descends vertically through 35 m. The lift has a mass of 0.52 tonnes and the two passengers each have a mass of 75 kg. Find the change in potential energy, stating whether it is an increase or a decrease.

5.2A.3 A car weighs 16 kN. The car slows down from a speed of 63 km h^{-1} to 45 km h^{-1}. Find the decrease in kinetic energy as a result of this deceleration, giving the answer in kilojoules (kJ).

5.2A.4 A bullet of mass 0.3 g is shot from a pistol at a speed of 400 m s^{-1}. Given that the bullet decelerates at 0.2 m s^{-2} as it travels 500 m, find the loss of kinetic energy.

5.2A.5 A particle of weight 45 N travels 23 m and gains 828 J of kinetic energy as it increases in speed from 19 m s^{-1} to V m s^{-1}.

(a) Find the value of V.

(b) Hence find the acceleration of the particle.

Exercise 5.3A · Conservation of energy and the work-energy principle

5.3A.1 A football of mass 0.4 kg is kicked vertically upwards from the ground. The initial speed of the ball is 18 m s^{-1}. Assume that there are no resistive forces.

(a) Find the initial kinetic energy of the football.

(b) Find the potential energy gained by the football when it reaches its maximum height.

(c) Find the height of the ball above the ground when it stops instantaneously.

5.3A.2 Starting from rest, a boy of mass 30 kg slides 5.4 m down a slide at a playground. The slide is rough, with a coefficient of friction of 0.15 and an angle of inclination, θ, of $\tan^{-1}(3/4)$.

(a) State the initial kinetic energy of the boy.

(b) Find the change in the potential energy of the boy.

(c) Find the normal reaction force exerted by the slide on the boy.

(d) Find the frictional force.

(e) Find the work done against friction.

(f) Find the speed of the boy at the bottom of the slide.

5.3A.3 A brick of mass 2.8 kg falls from the top of a block of flats. When it passes the tenth floor it has a speed of 11 m s^{-1}. When it passes the third floor it has a speed of 25 m s^{-1}. The floors are evenly spaced out.

(a) Find the change in the brick's kinetic energy between the tenth and third floors.

(b) By considering the loss of potential energy, find the distance between the fifth and seventh floors.

5.3A.4 A bowling ball has a mass of 1.6 kg. It rolls along a rough floor. Its initial velocity is 8 m s^{-1} and it rolls 48 m before coming to rest.

(a) Find the initial kinetic energy of the bowling ball.

(b) Find the work done against friction.

(c) Find the coefficient of friction between the ball and the floor.

5.3A.5 A bullet of mass 2.5 g is shot at 300 m s^{-1}. It immediately hits a wall and experiences a resistive force of 2.25 kN. Find how far into the wall the bullet penetrates.

5.3A.6 A pebble of mass 40 g is dropped from the top of a tower of height 60 m.

(a) By considering the energy of the pebble, find the speed of the pebble as it hits the ground.

(b) What assumption have you made in answering part (a)?

(c) Show that you get the same answer as in part (a) if you use the equation $v^2 = u^2 + 2as$.

5.3A.7 A particle of mass 0.7 kg is projected at 7 m s^{-1} up a rough plane inclined at 7° to the horizontal. The particle moves 18 m up the plane before coming to rest. Find the coefficient of friction between the particle and the plane.

5.3A.8 A boy and his bicycle have a total mass of 66 kg. Starting from rest, the boy freewheels d m down a road inclined at an angle of θ, where $\theta = \sin^{-1}(1/5)$. At the bottom of the slope the road becomes horizontal and the boy travels a further d m until he gets to a second slope at the same angle as the first, and he stops 6 m up the second slope. The boy experiences a constant resistive force of 12 N throughout.

(a) By considering energy changes, find the value of d.

(b) Find the speed at which the boy is travelling halfway along the horizontal section of road.

5.3A.9 Starting from point X, a particle P, of mass 2 kg, is projected from rest down a rough curved surface, XYZ, where path length XY = 20 m and YZ = 10 m. The vertical height of the surface between X and Y is 13 m and between Y and Z is H m. The particle experiences a constant resistive force of 4 N. The particle comes to rest at Z. Use the conservation of energy to find:

(a) the speed of P at point Y

(b) the value of H.

5.3A.10 A cart of mass 8.5 kg is projected at 14 m s^{-1} up a rough ramp inclined at an angle θ to the horizontal, where sin θ = 3/5. The ramp has a horizontal base length of 12 m. The cart stops instantaneously at the top of the ramp. By considering the conservation of energy, find the frictional force on the cart.

5.3A.11 A particle is projected up a rough plane inclined at an angle β to the horizontal, where sin β = 3/5. The height of the plane is 1.6 m. The coefficient of friction between the particle and the plane is 3/16. Given that the final velocity of the particle is given by v m s^{-1} and the initial velocity of the particle by (2v + 1) m s^{-1}, use the conservation of energy to find the value of v.

5.3A.12 A builder standing on the roof of a house dislodges a slate of mass 0.6 kg, which then slides down the roof and falls onto the ground and breaks. The roof is inclined at an angle of $\tan^{-1}(\sqrt{2} / 4)$ to the horizontal and is modelled as a rough surface with a coefficient of friction of $\sqrt{2} / 2$ between the roof and the slate. The vertical height of the roof is 3 m and the height of the whole house is 12 m. The speed of the slate when it hits the ground is 14 m s^{-1}. Find the distance the slate slides down the roof.

5.3A.13 A boy is skateboarding in a half pipe. The combined mass of the boy and his skateboard is 48 kg. From a standing start, he sets off from the edge of the deck and freewheels down a vertical section of 0.5 m, into a curved transition that is a quadrant of a circle of radius 3 m, across a flat horizontal bottom 6 m wide, and up a symmetric transition against a constant resistive force. The boy comes to rest at the top of the second transition.

(a) Find the magnitude of the resistive force.

(b) If the resistive force were 10 N instead, find the distance that the boy would travel up the second vertical section.

5.3A.14 A ball is released from point A at the top of a wooden track, AB, inclined at an angle of 20° to the horizontal. Section BD is also a wooden track, but horizontal. The coefficient of friction between the ball and each track is 0.3. The ball comes to rest at point C on the horizontal track, where BC = 6 m.

(a) Find the speed of the ball at B.

(b) Find the length of AB.

(c) Find the time taken for the ball to move from A to C.

Group Discussion · Racing car energy analysis

5.3D.1 A racing car requirement specifies that it can reach up to 300 km h^{-1} on a straight section of track. Write the maximum speed using SI units (m s^{-1}).

5.3D.2 State what physical factors influence the magnitudes of the forces acting upon the racing car during acceleration.

5.3D.3 Given that the car enters a straight at 120 km h^{-1} and experiences a constant resistance to motion of 1200 N, estimate the work that needs to be done by the engine of the car to get to 300 km h^{-1} within a distance of 100 m, for a car of mass 750 kg.

5.3D.4 Explain how your answer to question 3 would be affected if the straight section sloped uphill or downhill.

Exercise 5.4A · Power and acceleration

5.4A.1

(a) Find the power developed in a car travelling at 14 m s^{-1}, given that its engine is producing a driving force of 1500 N.

(b) A child is pushing a toy pushchair across the floor with a force of 10 N. The pushchair is moving at 0.2 m s^{-1}. Find the power developed by the child.

(c) The engine of a truck is working at a rate of 7200 W. Given that the truck is travelling at 54 km h^{-1}, find the driving force produced by the engine.

(d) The engine of a van is producing a driving force of 1.8 kN and working at a rate of 9.9 kW. Find the velocity of the van.

(e) The work done by a crane to lift a steel girder is 20 kJ. If it takes 10 s to lift the girder, find the power developed.

(f) An engine works at a rate of 6.1 kW for 29 s. Find the work done by the engine.

5.4A.2 A motorcycle of mass 250 kg is being ridden across a horizontal dirt track by a woman with a mass of 55 kg. The coefficient of friction between the motorcycle and the track is 2/5.

(a) Find the normal reaction force exerted on the motorcycle by the dirt track.

(b) Find the frictional force.

The woman rides 100 m in 5 s at the motorcycle's maximum velocity.

(c) Find the work done by the engine.

(d) Find the power developed by the motorcycle, giving the answer in kilowatts (kW).

(e) Find the acceleration of the motorcycle when it is moving at 12.5 m s^{-1} with the same power developed by the engine as in part (d).

5.4A.3 A car of mass 1200 kg is moving along a rough horizontal road. The engine is working at a rate of 7.2 kW. The coefficient of friction between the car and the road is 1/24. Find:

(a) the velocity when the car is accelerating at 1.25 m s^{-2}

(b) the velocity when the car is accelerating at 0.25 m s^{-2}

(c) the maximum velocity of the car.

5.4A.4 A car of mass 1.3 tonnes is driven up a rough road inclined at 6° to the horizontal. The engine of the car works at a rate of 14.8 kW. The coefficient of friction between the car and the road is 0.03.

(a) Find the maximum velocity of the car.

(b) Find the acceleration of the car when the velocity is 6 m s^{-1}, assuming that the engine works at the same rate.

5.4A.5 A minibus of mass 3200 kg is driven up a rough slope inclined at an angle of $\sin^{-1}(3/16)$ to the horizontal. It is subject to a constant resistance of 800 N. Given that the maximum velocity as it is driven up the slope is 1.8 m s^{-1}:

(a) find the power developed by the engine of the minibus.

The slope then becomes horizontal. Given that the power developed by the engine and the resistance are the same:

(b) find the initial acceleration of the minibus.

5.4A.6 A van is driven along a rough horizontal road at a maximum speed of 20 m s^{-1}. The engine of the van is working at 8.4 kW. The coefficient of friction between the van and the road is 1/25.

(a) Find the mass of the van.

(b) Find the work done by the van when it is driven for 30 seconds.

The road is now modelled as a rough slope inclined at 3° to the horizontal. The power developed by the engine and the coefficient of friction are assumed to be the same.

(c) Find the new maximum speed of the van.

(d) Find the acceleration of the van when it is moving up the hill at 5 m s^{-1}.

5.4A.7 A car of mass 800 kg tows another car of mass 1000 kg up a hill. The engine of the first car is working at a rate of 9.4 kW. The two vehicles are moving up the hill at 5 m s^{-1} and accelerating at 0.4 m s^{-2}. The cars are modelled as particles, each subject to a constant resistive force of 400 N, and the hill is modelled as a plane inclined at an angle of θ to the horizontal.

(a) Show that $\sin \theta = 0.02$.

(b) Hence find the maximum possible velocity, given that the power developed remains the same.

The vehicles become uncoupled when travelling at maximum velocity.

(c) Find the acceleration of the first car immediately after the uncoupling.

5.4A.8 Fen is riding her motorbike up a rough slope inclined at an angle of 4° to the horizontal. The combined mass of Fen and her motorbike is 180 kg. The power developed by the motorbike is 6.8 kW. When the motorbike is accelerating at 0.2 m s^{-2}, it has a velocity of 18 m s^{-1}. The coefficient of friction between the slope and the motorbike is given by μ.

(a) Find the value of μ.

When Fen is riding at maximum velocity up the same slope, she goes into neutral (so that the engine is no longer working).

(b) Use conservation of energy to find the further distance Fen travels up the slope before she stops instantaneously.

5.4A.9 A van of mass 1.6 tonnes is travelling along a horizontal road at V m s^{-1}. The frictional force is directly proportional to the velocity of the van. The power generated by the engine is 28.8 kW. When V = 12 m s^{-1}, the car accelerates at 1.26 m s^{-2}.

(a) Find a formula for the frictional force in terms of V.

(b) Find the value of V when the van is accelerating at 0.5 m s^{-2}.

5.4A.10 A car of mass 1200 kg travels along a straight road inclined at 3° to the horizontal. The car experiences a frictional force of 40V N, where V is the velocity of the car in m s^{-1}. Given that the power developed by the engine of the car in this situation is 75 kW:

(a) find the maximum velocity of the car.

The inclination of the road is now θ, where sin θ = 1/20. The power developed has reduced by 30 kW and the resistive force has fallen to 30V.

(b) Find the new maximum velocity of the car.

Activity · High speed power investigation

5.4D.1 Find the power developed by the engine of a racing car that has a maximum speed of 300 km h^{-1} against a resistance of 1200 N on a horizontal track.

5.4D.2 Find the power developed by the racing car in question 1 if the track is inclined upwards at:

(a) 2°

(b) 4°

(c) 6°.

5.4D.3 What engineering and safety considerations must be made as the engine power output increases significantly?

5.4D.4 If engine power output is limited to a maximum of 25 kW, what is the maximum resistance allowable on the horizontal track at 300 km h^{-1}?

5.4D.5 List three practical aerodynamic and mechanical steps an engineering team can take to reduce the motion resistance of a racing car.

Exam-style questions

R5.1 A coin is dropped into a well. The mass of the coin is 3 g and the depth of the well is 26 m.

(a) Find the loss of potential energy of the coin.

(b) Find the speed of the coin when it gets to the bottom of the well.

R5.2 A suitcase of mass 1.6 kg is projected 5 m along an airport floor until it comes to rest. Initially, the suitcase has 7.2 J of kinetic energy. The suitcase is modelled as a particle and the floor is modelled as rough and horizontal.

(a) Find the speed at which the suitcase is projected.

(b) State the work done against the resistive force.

(c) Find the magnitude of the resistive force.

R5.3 A remote-controlled car is driven across a tiled floor. The mass of the car is 1.3 kg. The car experiences a constant resistive force of 3 N. The battery in the car works at a rate of 7.5 W.

(a) Find the driving force when the car is driven at its maximum velocity.

(b) Find the maximum velocity of the remote-controlled car.

(c) Find the acceleration of the remote-controlled car when it is travelling at 1.2 m s^{-1}.

R5.4 A ball is thrown vertically upwards, with a speed of 24 m s^{-1}, from a height of 6 m above the ground. Assuming that all of the kinetic energy is converted into potential energy, find the maximum height reached by the ball above the ground.

R5.5 A bucket is accidentally dropped from the top of a building. When the bucket is h m above the ground it is falling at 22 m s^{-1}. When the bucket hits the ground it is falling at 28 m s^{-1}. The bucket gains 360 J of energy as it falls the final h m.

(a) Find the mass of the bucket.

(b) Find the value of h.

(c) Find the time taken for the bucket to hit the ground.

(d) Find the height of the building.

R5.6 A car that has run out of petrol is being towed by a breakdown truck along a straight horizontal road. The truck has mass 1600 kg and the car has mass 900 kg. The truck is connected to the car by a horizontal rope that is modelled as light and inextensible. The truck's engine provides a constant driving force of 2500 N. The resistances to motion of the truck and the car are modelled as constant and of magnitude 500 N and 300 N, respectively.

(a) Find the acceleration of the truck.

(b) Find the tension in the rope.

(c) Explain how you used the modelling assumption that the rope is inextensible.

When the breakdown truck is moving at 25 m s^{-1}, the rope breaks. The magnitude of the resistance to the motion of the car is assumed not to have changed.

(d) Use conservation of energy to find the distance travelled by the car after the rope breaks.

R5.7 A golf buggy of mass 225 kg is driven along a rough horizontal lawn at maximum speed. The engine of the buggy is working at 2.16 kW. The coefficient of friction between the golf buggy and the lawn is 0.16.

(a) Find the maximum speed of the golf buggy.

The golf buggy is now driven up a grass verge in the rough, inclined at an angle of 5° to the horizontal. The engine works at the same rate but the coefficient of friction is now 0.2.

(b) Find the maximum speed of the golf buggy on the grass verge.

R5.8 A ball of mass 250 g is released from rest at the top of a rough plane that is inclined at $\cos^{-1}$ 0.8 to the horizontal. The coefficient of friction between the particle and the plane is 3/7. The plane is of length 2.5 m. Find, at the point when the ball reaches the bottom of the plane:

(a) the potential energy lost by the ball

(b) the work done against friction by the ball

(c) the speed of the ball at the bottom of the plane.

R5.9 A toy robot of mass 2.5 kg is projected up a rough slope inclined at $\sin^{-1}(57/1625)$ to the horizontal. The robot comes to instantaneous rest after 6.5 m. The coefficient of friction between the robot and the slope is 5/232. Find:

(a) the work done against gravity

(b) the total work done.

The robot then slides back down the slope.

(c) Find the speed of the robot when it returns to its starting point.

R5.10 Two particles, A and B, approach each other at speeds of 2 m s^{-1} and 4 m s^{-1}, respectively. After the particles collide, they are travelling in different directions, A at 2.5 m s^{-1} and B at 3.5 m s^{-1}. The mass of particle B is 3 kg.

(a) Find the mass of particle A.

(b) Show that there is no loss of kinetic energy as a result of the collision.

R5.11 Two particles, P and Q, of masses 4 kg and 6 kg, respectively, are connected by means of a light inextensible string which passes over a smooth fixed pulley. Q is suspended 2 m above the ground. P is at rest on a rough slope 3 m from the pulley. The angle between the two parts of the string at the pulley is 60° (so the slope is inclined at 30° to the horizontal). The system is released from rest. Q falls to the ground. As P is pulled up the slope it is subject to a constant resistive force of 30 N.

(a) Use conservation of energy to find the velocity of Q when it hits the ground.

(b) Find the total distance travelled by P before it comes to instantaneous rest.

R5.12 A lorry pulls a cement mixer along a horizontal path. The coupling is inclined at 9° to the horizontal. The lorry and cement mixer have masses of 1600 kg and 800 kg, respectively. The coefficient of friction between each vehicle and the path is 1/16. The lorry sets off from a stationary position with a constant driving force of 3.3 kN.

(a) Find the initial acceleration of the lorry and cement mixer.

(b) Find the tension in the coupling.

After travelling 24 m, the coupling becomes disconnected.

(c) Find the power developed by the engine when the vehicles become uncoupled.

(d) Assuming that the power developed remains the same after the vehicles are uncoupled, find the maximum velocity of the lorry.

(e) Find the distance travelled by the cement mixer before it rolls to a stop.

R5.13 A stuntman on a bicycle starts his act from point X on a steeply curved concrete track, 30 m above the ground. The curved slope XY has length 50 m. At point Y, the track becomes horizontal for 138 m to point Z. At point Z, the track rises on a 30° incline. The combined mass of the stuntman and his bicycle is 200 kg. The bicycle is subject to a constant resistance of 232 N throughout. From a standing start at X, he freewheels down the slope.

(a) Find the speed at which the stuntman is travelling at Y.

(b) Find the speed at which the stuntman is travelling at Z.

(c) Find the distance the stuntman travels up the 30° slope before coming to rest.

R5.14 A tractor of weight 25 000 N pulls a combine harvester up a rough slope inclined at 10° to the horizontal. The coefficient of friction between the slope and each vehicle is 0.125. The engine of the tractor is working at a rate of 48 kW. The maximum speed of the vehicles is 4.2 m s^{-1}.

(a) Find the weight of the combine harvester.

(b) Find the tension in the coupling.

Suddenly the vehicles become uncoupled.

(c) Find the time it takes for the combine harvester to come to instantaneous rest.

R5.15 Angelene skateboards along a track. The track starts and finishes with straight slopes AB and CD, each inclined at θ to the horizontal, where sin θ = 3/5, and each 10 m long. Between the two slopes is a horizontal section BC of length d m. The coefficient of friction between the skateboard and the track is 1/10 for all three sections. Angelene starts from rest at A and freewheels all the way. She has just enough motion at C to continue down the final section CD.

(a) Find the value of d.

Given that Angelene has 3.12 kJ of kinetic energy at D:

(b) find the combined mass of Angelene and her skateboard

(c) find the speed at which Angelene is moving when she reaches D.

R5.16 Two particles, P and Q, of masses 9 kg and 7 kg, respectively, are connected by a light inextensible string. The particles are suspended vertically with P above Q, and Q is 5 m above the ground. A force of 200 N is applied vertically upwards on P such that P is raised 8 m from rest.

(a) Find the work done by the 200 N force.

(b) Find the work done against gravity.

(c) Find the speed of the particles when they have been raised 8 m.

When the particles have been raised 8 m, the string snaps.

(d) Find the speed of Q when it hits the ground.

R5.17 A car of mass 1800 kg is moving along a horizontal road. The power developed by the engine of the car is 60.2 kW. The resistive force experienced by the car as it moves along the road is given by $(28v + 20)$ N, where v m s^{-1} is the velocity of the car. When the car is accelerating at 0.4 m s^{-2}, the velocity is given by V m s^{-1}.

(a) Show that $7V^2 + 185V = 15050$.

(b) Hence find the value of V.

(c) Show that the maximum velocity of the car is about 11 m s^{-1} faster than V.

R5.18 A particle of mass 6.8 kg moves 15 m down a plane from A to B, along a horizontal path of length d_1 m to C, then d_2 m up a second plane until it comes to instantaneous rest at D. The particle is subject to a constant frictional force of F N throughout. The first and second planes are inclined at respective angles α and β to the horizontal, where $\sin \beta = 13/85$. The particle has an initial gravitational potential energy of 480 J relative to line BC. The total distance travelled by the particle is 65 m and d_1 and d_2 are in the ratio 3 : 1.

(a) Find the value of $\sin \alpha$, giving the answer as a simplified fraction.

(b) Find the gravitational potential energy of the particle at D.

(c) Find the value of F, correct to 1 decimal place.

(d) Find the velocity of the particle at C.

R5.19 The heights of the front and back of a shed are h m and (3/2)h m, respectively. A tile slides h m down the roof from the back wall to the front wall before falling vertically from height h to the ground. The roof is rough and the coefficient of friction between the roof and the tile is given by $\sqrt{3}$ / 3. Use conservation of energy to show that the speed of the tile when it hits the ground is given by $\sqrt{(2gh)}$.

Mathematics in life and work · Racing car circuit dynamics

M5.1 A racing car team prepares a car for a track segment consisting of a 140 m curved downhill slope of vertical height 20 m, followed by a straight slope inclined at 3° to the horizontal. The mass of the car is 720 kg (including driver and fuel). The car experiences a constant resistive force of 2400 N on the curved section. At the top of the curved slope, the car travels at 288 km h^{-1}. No driving force is applied on the downhill curved slope. Find the speed of the car at the bottom of the curved slope.

M5.2 When the car reaches the bottom and begins to move up the 3° straight slope, its initial acceleration is 0.4 m s^{-2}. The coefficient of friction on the straight slope is 0.2. Find the power developed by the engine at the beginning of the straight slope.

M5.3 The engine continues to work at this constant power as it climbs the slope. Find the maximum steady speed of the car on the slope.

Check yourself

C5.1 A sled of mass 40 kg is pulled 25 m across a horizontal frozen pond by a constant force of 60 N acting at an angle of 28° above the horizontal. A constant resistance of 15 N opposes the motion. Starting from rest:

(a) Calculate the work done by the pulling force.

(b) Calculate the work done against the resistance force.

(c) Using the work-energy principle, find the speed of the sled after travelling the 25 m.

C5.2 A box of mass 15 kg is projected up a rough ramp inclined at 20° to the horizontal with an initial speed of 8 m s^{-1}. The coefficient of friction between the box and the ramp is 0.25.

(a) Calculate the work done against friction as the box moves a distance d m up the ramp.

(b) Calculate the gain in gravitational potential energy in terms of d.

(c) Use conservation of energy to determine the distance d the box travels before coming to instantaneous rest.

C5.3 A roller-coaster car of mass 600 kg starts from rest at point A, which is 28 m above the ground. It travels down a curved frictionless track to ground level B, then climbs a second hill to point C at a height of 18 m above the ground.

(a) Find the speed of the car at point B.

(b) Find the speed of the car at point C.

(c) In a real run, resistance forces do 24 kJ of work on the car between A and C. Find the actual speed of the car at point C.

C5.4 A car of mass 1100 kg travels along a straight horizontal road. The car's engine works at a constant rate of 33 kW. The non-gravitational resistance to motion is constant and equal to 550 N.

(a) Find the maximum steady speed of the car on the horizontal road.

(b) Find the acceleration of the car at the instant when its speed is 20 m s^{-1}.

C5.5 A van of mass 1500 kg climbs a straight hill inclined at an angle θ to the horizontal, where sin θ = 1/20. The resistance to motion is constant at 600 N. The engine of the van works at a steady rate of 45 kW.

(a) Find the driving force when the van is moving up the hill at a steady maximum speed.

(b) Calculate this maximum steady speed.

(c) When travelling at this maximum speed, the van reaches the crest of the hill and continues onto a level horizontal stretch. If the power and resistance remain unchanged, find the instantaneous acceleration of the van immediately after reaching the level road.

Chapter quiz

Q1 A force of 50 N pulls a box 12 m along a floor at 60° above the horizontal. What is the work done?

A	B	C	D
600 J	300 J	520 J	150 J

Q2 A 25 kg crate is lifted vertically through 8 m at constant speed ($g = 10$ m s^{-2}). Find work done against gravity.

A	B	C	D
200 J	2000 J	250 J	1600 J

Q3 What is the kinetic energy of a 1200 kg car travelling at 20 m s^{-1}?

A	B	C	D
24 kJ	240 kJ	480 kJ	120 kJ

Q4 A 4 kg particle accelerates from 5 m s^{-1} to 11 m s^{-1}. What is the gain in kinetic energy?

A	B	C	D
192 J	96 J	144 J	384 J

Q5 A 0.5 kg ball drops from rest through a vertical height of 20 m ($g = 10\ m\ s^{-2}$). Find its speed just before impact.

A	B	C	D
10 m s^{-1}	14.1 m s^{-1}	20 m s^{-1}	40 m s^{-1}

Q6 A block slides down a slope losing 80 J of GPE. If 30 J of work is done against friction, find its gain in KE.

A	B	C	D
110 J	50 J	2400 J	25 J

Q7 A car engine delivers 45 kW of power at a constant speed of 25 m s^{-1}. What is the driving force produced?

A	B	C	D
1125 N	1800 N	1.8 N	3600 N

Q8 A car faces a constant resistance of 600 N on level ground. What is its maximum speed when working at 30 kW?

A	B	C	D
20 m s^{-1}	50 m s^{-1}	18 m s^{-1}	5 m s^{-1}

Q9 A car of mass 800 kg has engine power 24 kW. When moving at 20 m s^{-1} against resistance 400 N, find its acceleration.

A	B	C	D
1.0 m s^{-2}	1.5 m s^{-2}	0.5 m s^{-2}	2.0 m s^{-2}

Q10 An electric hoist lifts a 150 kg load through 12 m in 6 s ($g = 10\ m\ s^{-2}$). What is the average power developed?

A	B	C	D
300 W	1800 W	3000 W	18 000 W

Q11 A 1000 kg car climbs a slope ($\sin\theta = 0.05$) at steady 15 m s^{-1} against resistance 300 N ($g = 10$). Find engine power.

A	B	C	D
12 kW	7.5 kW	4.5 kW	15 kW

Q12 When a car freewheels down a rough hill from rest, which statement correctly expresses the work-energy relation?

A	B	C	D
Gain in KE = Loss in GPE + Work against friction	Loss in GPE = Gain in KE + Work against friction	Gain in KE = Work against friction − Loss in GPE	Loss in GPE = Work against friction − Gain in KE

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/m1-5-energy-work-and-power/