M1·4

Newton’s laws of motion

CHAPTER ASSIGNMENT

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. State appropriate units and degrees of accuracy. Take g = 10 m s^{-2} unless specified otherwise.

Practice questions from *Cambridge International AS & A Level Mathematics: Mechanics Student’s Book* (Collins, Tom Andrews and Michael Kent), Chapter 4 (Newton’s laws of motion), Exercises 4.1A, 4.2A, 4.3A, 4.4A, 4.4B, 4.5A and Exam-style questions, for classroom use. Where a question needs a diagram, the diagram is described in words or given as a table. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 4.1A · Newton's laws of motion

4.1.1 (a) Find the magnitude of the resultant force acting on a particle of mass 800 g that would produce an acceleration of 3.1 m s^{-2}.

(b) A particle of mass m kg has a deceleration of 4 m s^{-2} when acted upon by a resultant force of 5.2 kN. Find the value of m.

4.1.2 (a) Find the weight of a dog of mass 8.4 kg on the Earth's surface.

(b) An astronaut weighs 735 N on the surface of Earth. Find the weight of the astronaut on the surface of Mars (g = 3.75 m s^{-2}).

4.1.3 A car travelling along a straight horizontal road produces a driving force of 1.27 kN whilst experiencing a frictional force of 190 N. The car takes 15 s to accelerate uniformly from 4 m s^{-1} to 22 m s^{-1}. Find the mass of the car.

4.1.4 A particle at rest 2 m from the edge of a horizontal table is subjected to a thrust of X N. The coefficient of friction is 1/3. The mass of the particle is 75 kg. Find the value of X given that:

(a) the particle does not accelerate

(b) the particle accelerates at 2 m s^{-2}

(c) the particle falls off the edge of the table after 0.5 seconds.

4.1.5 A minibus of mass 1500 kg produces a driving force of 1125 N. Given that the initial velocity of the minibus is 4 m s^{-1}, find the time it takes the minibus to travel 385 m along a straight horizontal road. The coefficient of friction between the minibus and the road is 0.035.

4.1.6 A car of mass 960 kg accelerates from 15 m s^{-1} to 30 m s^{-1} along a straight 400 m stretch of a horizontal motorway at a constant rate. During this acceleration, the engine produces a driving force of 1.24 kN. The motorist then spots a fault on the dashboard and decides to pull over onto the hard shoulder, removing the driving force. The car then decelerates at a constant rate with a braking force of 500 N. Assuming that the frictional force is constant throughout, how much further does the car travel before it comes to rest?

4.1.7 Starting from rest, a 1.6 tonne coach travels along a straight horizontal lane. The engine exerts a driving force of 5.9 kN. The coach experiences a uniform frictional force of 1.1 kN. After 11 s, the driver spots a roadblock 260 m ahead. He immediately decelerates at a constant rate, reducing the driving force to zero and applying a braking force, and comes to rest 20 m before the roadblock. Find the braking force required to bring the coach to a halt.

4.1.8 A remote-controlled truck moves along a rough horizontal floor. The coefficient of friction between the floor and the truck is 3/8 and the driving force is 54 N. When the truck is placed onto a different horizontal surface, for which the coefficient of friction between the surface and the truck is 2/3, it requires a driving force of 89 N to maintain the same acceleration.

(a) Find the mass of the truck.

(b) Find the acceleration.

4.1.9 A trolley is pulled across a rough horizontal car park by a rope inclined at an angle α to the horizontal. When $\cos \alpha = 20/29$, the coefficient of friction between the trolley and the car park is 17/40 and the trolley accelerates at 0.2 m s^{-2}. When instead $\sin \alpha = 20/29$, the normal reaction force between the trolley and the car park is 180 N. Assuming that the tension is constant:

(a) find the tension in the rope

(b) find the mass of the trolley.

4.1.10 A crate of mass 20 kg is pulled across a rough horizontal yard by means of a light handle inclined at angle θ to the horizontal, where $\sin \theta = 3/5$. The coefficient of friction between the crate and the surface of the yard is 0.25. The tension in the handle is 60 N. The initial velocity of the crate is 4 m s^{-1}. The total time taken to pull the crate 1.44 km is given by t s.

(a) Show that $7t^2 + 160t = 57\,600$.

(b) Hence find the value of t.

Exercise 4.2A · Vertical motion

4.2.1 A ball of mass 120 g is dropped from the top of a tower of height 25 m. The ball experiences an air resistance of 0.03 N as it falls. Find how much longer it takes for the ball to hit the ground than if air resistance were ignored.

4.2.2 A toy of mass 350 g is dropped in a swimming pool and hits the surface of the water before falling vertically to the bottom of the pool. The depth of the pool is 1.9 m. The toy experiences a resistance to its motion of 2.8 N. The initial velocity of the toy as it hits the water is 3.7 m s^{-1}. Find the time it takes the toy to reach the bottom of the pool.

4.2.3 A box of mass M kg is held at the top of a vertical rod. When the box is lowered vertically with an acceleration of 0.4 m s^{-2}, the compression in the rod is 12 N. Find the compression in the rod when the box is raised vertically with an acceleration of 0.4 m s^{-2}.

4.2.4 A 20 kg rock is pulled vertically upwards by a vertical cable from a speed of 2 m s^{-1} to a speed of 5 m s^{-1}. The tension in the cable is 250 N. Calculate how far the rock is pulled.

4.2.5 A 3 N weight at rest on the floor is attached to a vertical string, which pulls it vertically upwards. When the weight is 2.5 m above the floor, it is moving at 4 m s^{-1}.

(a) Find the tension in the string.

The string is then released.

(b) Find the maximum height achieved by the weight.

(c) Find the velocity of the weight when it returns to the floor.

4.2.6 A 28 kg block is projected vertically upwards from the ground at 7 m s^{-1}. When the block reaches a height of 2 m, it experiences a force of 420 N vertically upwards until it is 18 m above the ground. The force is then removed. Assume that air resistance can be ignored.

(a) Find the total time taken for the block to reach the height of 18 m.

(b) Find the speed of the block when it returns to the ground.

4.2.7 A diver of mass 50 kg drops from the 10 m board at the swimming pool. As the diver drops, she experiences air resistance of magnitude 10 N. When she goes under water, the water resistance is of magnitude 2500 N. Let g be 9.8 m s^{-2}.

(a) Find the maximum distance the diver travels under the surface of the water.

A second diver, of mass 60 kg, copies the first diver. Assume that the resistances are the same.

(b) How much further does the second diver travel under the surface of the water?

4.2.8 Two rocks fall from a cliff, 45 m above a pool of mud. When the rocks are in the mud, they experience a resistive force of 3.25 kN. One rock is 6 kg heavier than the other and the final position of this rock is 25 cm lower than for the other. Assume that air resistance can be ignored. Find the mass of the lighter rock.

Exercise 4.3A · Inclined planes

4.3.1 A tile of mass 900 g slides down a sloped roof. The roof is inclined at 26° to the horizontal. Given that the coefficient of friction between any tile and the roof is 0.35:

(a) find the acceleration of the tile.

A second tile of mass 800 g slides down the roof.

(b) State the acceleration of the second tile.

4.3.2 A basket of mass 4 kg is pushed up a smooth plane by a force of 26 N, along the line of greatest slope. The plane is inclined at an angle θ to the horizontal. If the basket is accelerating at 0.5 m s^{-2}, find the value of $\sin\theta$.

4.3.3 A ball of mass 3 kg is released from rest on a rough surface inclined at $\sin^{-1} 0.6$ to the horizontal. After 2.5 s, the ball has acquired a velocity of 4.9 m s^{-1} down the surface. Find the coefficient of friction between the body and the surface.

4.3.4 A body of mass 5 kg is initially at rest at the bottom of a rough inclined plane of length 6.3 metres. The plane is inclined at 30° to the horizontal. The coefficient of friction between the body and the plane is $1/(2\sqrt{3})$. A constant horizontal force of magnitude $35\sqrt{3}$ N is applied to the body causing it to accelerate up the plane. Find:

(a) the time taken for the body to reach the top of the plane

(b) the speed of the body on arrival at the top of the plane.

4.3.5 A box of mass 5 kg is pulled along rough horizontal ground by means of a rope inclined at $\sin^{-1}(44/125)$ to the ground. When the tension in the rope is 10 N the box is moving at constant speed.

(a) Calculate the coefficient of friction between the box and the ground.

(b) Calculate the acceleration of the box if the tension is increased to 15 N.

4.3.6 A particle of mass 2 kg rests in limiting equilibrium on a plane inclined at 25° to the horizontal. The angle of inclination is decreased to 20° and a force of magnitude 20 N is applied up a line of greatest slope.

(a) Find the particle's acceleration.

When the particle has been moving for 2 seconds the force is removed.

(b) Determine the furthest distance the particle will move up the plane.

4.3.7 A body of mass 500 g is placed on a rough plane which is inclined at 40° to the horizontal. The coefficient of friction between the body and the plane is 0.6.

(a) Find the maximum value of the frictional force.

(b) Show that motion will occur, stating the acceleration of the body.

4.3.8 A horizontal force of 4 N is just sufficient to prevent a block of mass 2 kg from sliding down a rough plane inclined at 23° to the horizontal. If the 4 N force is replaced by a force of X N, then the block is on the point of moving up the plane.

(a) Find the value of X.

(b) Find the acceleration if the X N force acts parallel to the slope rather than horizontally.

4.3.9 A particle of mass 2 kg slides down two rough planes. The first plane is inclined at α to the horizontal, where $\sin \alpha = 3/5$, and the second plane is inclined at β to the horizontal, where $\sin \beta = 7/25$. The coefficient of friction, μ between the particle and each plane is the same. Given that the acceleration of the particle on the first plane is ten times greater than that on the second, find the value of μ.

4.3.10 A block is projected up a rough plane at 6 m s^{-1}, coming to instantaneous rest after 8 m. The plane is inclined at 7.5° to the horizontal.

(a) Find the coefficient of friction between the block and the plane.

(b) Find the total time taken for the block to return to its original position.

Exercise 4.4A · Connected particles

4.4A.1 Two particles, G and H, are at rest on a smooth horizontal table and connected by a light inextensible string. H is pulled by a horizontal force of 3 N, which causes the particles to move. The mass of G is 150 g and the mass of H is 350 g. Find the tension in the string and the acceleration of G.

4.4A.2 A particle P, of mass 4m kg, is suspended from a vertical rope. A particle Q, of mass 3m kg, is suspended from particle P by another vertical rope. A vertical force, X N, is applied to P such that the particles accelerate upwards at (4/7)g. Find the tension in the rope connecting P and Q, and the value of X, giving the answers in terms of m and g.

4.4A.3 A car of mass 300 kg is towing a caravan of mass 840 kg along a straight horizontal stretch of motorway. The engine of the car is exerting a driving force of 1.71 kN. Both vehicles experience resistive forces that are proportional to their masses. Find the tension in the tow bar.

4.4A.4 Two particles of masses 7 kg and 4 kg on a rough horizontal table are connected by a light inextensible 50 cm string. The heavier particle is projected away from the lighter particle by a force of 35 N. The numerical value of the coefficient of friction between each particle and the table is one-twentieth of its mass. After 4 seconds, the heavier particle hits a wall and does not bounce back.

(a) Does the lighter particle collide with the heavier particle?

(b) If it does, how much longer does it take the lighter particle to reach the heavier particle; if it does not, how far apart do they end up?

4.4A.5 A particle, X, of mass 5 kg, is suspended from a vertical string. A second particle, Y, of mass 3 kg, is suspended from X by a vertical cord. A force of 100 N is applied to the string and the two particles accelerate upwards. When the particles are travelling at 9 m s^{-1}, the cord snaps. How long does it take Y to return to its original position?

4.4A.6 A minibus of mass 950 kg is towing a small trailer along a straight horizontal road. The resistances to motion are 475 N and 25 N, respectively. While the minibus is increasing in speed from 12 m s^{-1} to 32 m s^{-1} in 8 s, the driving force of the minibus is 3 kN. When the minibus reaches 32 m s^{-1}, the minibus and trailer become uncoupled.

(a) Find the mass of the trailer.

(b) Show that the trailer travels 1024 m before it comes to rest.

4.4A.7 A car of mass M kg is towing a caravan of mass 750 kg along a straight horizontal road. The resistance on the car is X N and the resistance on the caravan is 400 N. When the engine of the car exerts a driving force of 1450 N, the car accelerates at 0.6 m s^{-2} but when the engine of the car exerts a driving force that is 1 kN larger, the car accelerates at 1.4 m s^{-2}.

(a) Find the value of M.

(b) Find the value of X.

4.4A.8 Two particles, X and Y, are connected by a light string, with X vertically above Y. The masses of X and Y are 6 kg and 14 kg, respectively. X experiences a force of 208 N pulling it upwards for 5 s from rest. At this point, the string snaps. Find the further time it takes for Y to reach its maximum height.

4.4A.9 A car of mass 900 kg pulls a caravan of mass 1200 kg up a rough hill inclined at 7° to the horizontal. The acceleration of the car as it drives up the hill is 0.65 m s^{-2}. After a certain time, the hill increases in steepness so that it is then inclined at 10° to the horizontal, and under these conditions the acceleration of the car reduces to 0.15 m s^{-2}. The driving force exerted by the engine of the car is the same for each gradient. The coefficient of friction between each vehicle and the hill is constant throughout.

(a) Find the coefficient of friction.

(b) Find the driving force exerted by the engine of the car.

4.4A.10 A small train is made from three sections, A, B and C, such that the engine A pulls the two carriages B and C along a horizontal track. The mass of B is M kg. B is 40% lighter than A and 50% heavier than C. The coefficient of friction between each section and the track is 1/10. The driving force exerted by the engine is 8.4 kN. Having travelled 2.79 km in 3 minutes at a constant acceleration and reaching a velocity of 20 m s^{-1}, C becomes uncoupled from the rest of the train.

(a) Find the value of M.

(b) Find the tension in the coupling between A and B immediately after C is uncoupled.

(c) Find the time taken for C to stop after the uncoupling.

Exercise 4.4B · Connected particles in lifts

4.4B.1 A teenager standing in a lift has a mass of 55 kg. Find the reaction force exerted on the teenager by the lift when the lift is:

(a) accelerating upwards at 2.4 m s^{-2}

(b) accelerating downwards at 1.6 m s^{-2}

(c) decelerating upwards at 0.6 m s^{-2}.

4.4B.2 A dog of mass 25 kg sits in a lift of mass 140 kg. The lift is descending at 1.8 m s^{-2}.

(a) Find the reaction force exerted by the lift on the dog.

(b) Find the tension in the lift cable.

4.4B.3 Two adults, of masses 75 kg and 65 kg, enter a lift. The lift has a mass of 550 kg. The tension in the lift cable is 7616 N. Find the time it takes for the lift to ascend 24 m.

4.4B.4 Five people, of average mass 66 kg, travel upwards in a lift of mass 280 kg, at 0.2 m s^{-2}.

(a) Find the tension in the lift cable.

When a sixth person joins the lift, the average mass increases by 4 kg. Given that the acceleration decreases by 0.05 m s^{-2}:

(b) find the reaction force exerted on the sixth person by the floor of the lift

(c) find the new tension in the lift cable.

4.4B.5 A girl and her father stand in a lift which is travelling downwards at (1/2) m s^{-2}. The father weighs twice as much as the girl and the lift weighs 100 kg more than the father. The tension in the lift cable is 2.66 kN. Find the reaction force exerted by the lift floor on the father.

4.4B.6 Liu and Zhao, whose masses are in the ratio 2 : 3, respectively, take separate journeys in a lift of mass 360 kg. When Liu ascends at a m s^{-2}, the tension in the lift cable is 4.73 kN. When Zhao descends at (a – 0.25) m s^{-2}, the reaction force exerted on him by the lift is 1.14 kN.

(a) Find the value of a.

Zhao and Liu now take the lift at the same time, ascending at 0.6 m s^{-2}.

(b) Find the tension in the lift cable.

4.4B.7 A lift of mass 400 kg has a notice stating that the maximum load is 160 kg. When the lift ascends with a load of this mass, the lift accelerates at 0.75 m s^{-2}. However, a caretaker carelessly puts a 200 kg crate in the lift whilst the lift is on the ground floor, without checking the notice. After the lift has been ascending for 30 s, the cable snaps and the lift returns to the ground floor. Assuming that the tension in the cable before it snaps is the same as when the load was 160 kg, how long after the cable snaps does it take for the lift to return to the ground floor?

4.4B.8 A hotel has a lift of mass 280 kg.

(a) Margarita gets into the lift. As the lift ascends, the tension in the lift cable is 4320 N and Margarita experiences a reaction force of 960 N from the floor of the lift. Find:

(i) Margarita's mass

(ii) the acceleration of the lift.

(b) Margarita gets out of the lift and Guillaume and Josef get in. Now, as the lift ascends, the tension in the lift cable is 5100 N. Guillaume and Josef experience reaction forces of 700 N and 900 N, respectively, from the floor of the lift. Find:

(i) Guillaume's mass

(ii) Josef's mass

(iii) the acceleration of the lift.

Exercise 4.5A · Pulleys

4.5.1 Particles G and H, of masses 3m kg and 2m kg, are connected by a light inextensible string with G at rest on a smooth table. The string passes over a smooth fixed pulley at the edge of the table with H hanging freely beneath the pulley. The system is released from rest.

(a) Find the force exerted on G by the table in terms of m and g.

(b) Find the acceleration of G (or of H) in terms of g.

(c) Find the tension in the string in terms of m and g.

4.5.2 Two particles, of masses 3 kg and 4 kg, respectively, are connected by an inextensible string via a smooth fixed pulley, both hanging vertically. The system is released from rest.

(a) Find the acceleration of either particle in terms of g.

(b) Find the tension in the string in terms of g.

(c) Find the difference in height between the particles after 1.4 s.

4.5.3 Particles P_1 and P_2 are connected by a light inextensible cord passing over a smooth fixed pulley. P_1 has a mass of 4 kg and P_2 is heavier than P_1. When the system is released from rest, the acceleration of each particle is equal to $(1/5)g$ m s^{-2}.

(a) Find the mass of P_2.

(b) Find the force exerted on the pulley.

4.5.4 A particle J sits on a rough horizontal table connected by a light inextensible string via a smooth fixed pulley at the edge of the table to a particle K, which is suspended 3 m above the ground. Particles J and K have masses 2 kg and 3.2 kg, respectively. The coefficient of friction between particle J and the table is 0.3. K is released from rest with the string taut.

(a) Find the friction experienced by particle J.

(b) Find the acceleration of J and K.

(c) Find the tension in the string.

(d) Assuming that J does not meet the pulley, how far does it travel altogether before it stops?

4.5.5 Particles A and B, of masses 3 kg and 7 kg, are connected by a light inextensible wire that passes over a smooth pulley. A is at rest on a smooth table 5 m from the pulley while B is hanging freely 2 m above the ground. When the system is released, how long does it take A to reach the pulley?

4.5.6 Two particles of masses x kg and y kg are connected by a light inextensible string passing over a smooth fixed pulley. Both particles are hanging freely. Show that when the system is released from rest, the tension is given by 2xyg / (x + y).

4.5.7 A ball of mass 500 g is suspended vertically from a smooth fixed pulley at the edge of a rough table. The ball is connected via a light inextensible string and the pulley to a flowerpot of mass 300 g on the table. The coefficient of friction between the flowerpot and the table is 2/3. The ball is released and the string snaps after 0.2 s. Assuming that the flowerpot does not reach the pulley, find the total time for which the flowerpot is in motion.

4.5.8 Two particles, X and Y, of masses 700 g and 800 g, respectively, are connected by a light inextensible cord. X is at rest on a smooth slope inclined at an angle θ to the horizontal, where sin θ = 1/4. The cord passes over a smooth light pulley fixed at the top of the slope. Y hangs freely from the pulley. The system is released from rest with the string taut. Calculate:

(a) the acceleration of Y

(b) the tension in the cord.

Y hits the ground after 3 s and remains at rest on the ground without bouncing.

(c) Find the distance travelled by X before Y hits the ground.

In the subsequent motion, X does not reach the pulley.

(d) Find the further distance travelled by X before it comes to instantaneous rest.

4.5.9 Two rocks, P and Q, are connected by an inextensible rope passing over a smooth pulley. Q is 3 kg heavier than P. Given that the tension in the rope is 45.5 N, find the mass of each rock and the acceleration of the system.

4.5.10 Two particles, A and B, of masses 1.8 kg and 5.2 kg, respectively, are connected by a light inextensible string passing over a fixed smooth light pulley. Particle A is placed on a rough plane inclined at 30° to the horizontal. The coefficient of friction between particle A and the plane is $\sqrt{3} / 9$. Particle B hangs freely below the pulley. The particles are released from rest with the string taut.

(a) Show that the acceleration of the particles when released is given by $(4/7)g$ m s^{-2}.

(b) Calculate the tension in the string.

Particle B strikes the floor after 2 s.

(c) Find the total time from when the particles were released until A comes to instantaneous rest (assuming that it does not reach the pulley).

Exam-style questions · Newton's laws review

R4.1 A box of mass M kg is pushed along a rough horizontal table by a horizontal force of 24 N. The coefficient of friction between the box and the table is 0.4.

(a) Find the magnitude of the frictional force in terms of M and g.

The box is accelerating at 2 m s^{-2}.

(b) Write an equation of motion for the box.

(c) Find the value of M.

R4.2 A girl is travelling down a rough slope in a go-kart. The combined mass of the girl and the go-kart is 50 kg. The slope is inclined at an angle of 30° to the horizontal. The coefficient of friction between the go-kart and the slope is 1/6.

(a) Find the magnitude of the reaction force exerted on the girl and the go-kart by the slope.

(b) Find the acceleration of the go-kart down the slope.

(c) Given that the go-kart started from rest, how long does it take the girl to travel 15 m down the slope?

R4.3 A particle A, of mass 5 kg, rests on a smooth horizontal table. Particle A is attached to one end of a light inextensible string which passes over a smooth pulley fixed at the edge of the table. The other end of the string is attached to a particle B, of mass 3 kg, which hangs freely below the pulley, 1.3 m above the ground. The system is released from rest with the string taut. In the subsequent motion, particle A does not reach the pulley before B reaches the ground.

(a) Find the tension in the string before B reaches the ground.

(b) Find the time taken by B to reach the ground.

R4.4 A student is set the following question: Two particles are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. The particles are held in position with the string taut and the hanging parts of the string vertical. The particles are then released from rest. The particles have masses of 6 kg and m kg, where $m < 6$, and the initial acceleration of each particle has a magnitude of $(2/3)g$ m s^{-2}. Find: (i) the tension in the string immediately after the particles are released, (ii) the value of m.

The student wrote:

Left: $T - 6g = 6 \times (2/3)g \Rightarrow T = 10g = 100$ N

Right: $mg - T = (2/3)mg \Rightarrow mg - 100 = (2/3)mg \Rightarrow (1/3)mg = 100 \Rightarrow m = 30$ kg.

Assess the student's solution and make corrections where necessary.

R4.5 Two particles, A and B, are connected by a light inextensible vertical cable with A above B. The mass of A is 500 g and the mass of B is 200 g. Initially, the system is at rest. A vertical force of X N is applied to A, which raises the particles 24 m in 4 s. Find the magnitude of X.

R4.6 A man uses the lift in a hotel. The lift starts from rest on the ground floor and moves vertically upwards. The lift initially accelerates with a constant acceleration of 0.8 m s^{-2} for 15 s. It then decelerates for 10 s before coming to rest. The mass of the man is 90 kg and the mass of the lift is 160 kg. The lift is pulled up by means of a vertical cable attached to the top of the lift. By modelling the cable as light and inextensible, find:

(a) the tension in the cable when the lift is accelerating

(b) the magnitude of the force exerted by the lift on the man when the lift is decelerating.

(c) Explain how you have used the assumption that the cable is light.

R4.7 A car of mass 400 kg pulls a horsebox of mass 600 kg up a rough hill inclined at $\sin^{-1} 0.2$. The resistances to motion are 400 N for the car and 600 N for the horsebox. The car exerts a driving force of 3.2 kN.

(a) Find the acceleration.

The car becomes uncoupled from the horsebox after travelling 90 m.

(b) Find the new acceleration of the car.

(c) Show that the horsebox travels 6 m after the uncoupling.

R4.8 A particle P, of mass 7 kg, is projected vertically upwards from a point A with speed 22 m s^{-1}. The point A is 12.25 m above horizontal ground. The particle P moves freely under gravity until it reaches the ground with speed V m s^{-1}.

(a) Find the value of V.

The ground is soft and, after it reaches the ground, P sinks vertically downwards into the ground before coming to rest. The ground exerts a constant resistive force of magnitude 3000 N on P.

(b) Find the vertical distance that P sinks into the ground before coming to rest.

R4.9 Two particles, P_1 and P_2, of mass 4.5 kg and 5.5 kg, respectively, initially at rest, are joined by a light horizontal rod. A constant force of magnitude 7 N is applied to P_2 and the system moves under the action of this force for 10 s. During the motion, the resistance to the motion of P_1 has a constant magnitude of 2 N and the resistance to the motion of P_2 has a constant magnitude of 3 N. Find:

(a) the acceleration of the particles as the system moves under the action of the 7 N force

(b) the speed of the particles after 10 s

(c) the tension in the rod as the system moves under the action of the 7 N force.

After 10 s, the 7 N force is removed and the system decelerates to rest. The resistances to motion are unchanged. Find:

(d) the distance moved by P_1 as the system decelerates

(e) the thrust in the rod as the system decelerates.

R4.10 A van pulls a cart along a rough horizontal road. The mass of the van is 900 kg and the mass of the cart is 500 kg. The van and cart experience frictional forces of 800 N and 400 N, respectively. The tow bar coupling the van and cart together is inclined at 12° to the horizontal. The van exerts a driving force of 2.25 kN.

(a) Find the acceleration of the van.

(b) Show that the magnitude of the tension in the tow bar is 790 N, correct to 2 significant figures.

R4.11 A car of mass 750 kg is pulling a trailer of mass 250 kg along a straight horizontal road by means of a tow bar. The resistances to motion of the car and the trailer are 300 N and 100 N, respectively. The engine of the car is exerting a constant driving force of 1.5 kN. The car and trailer are modelled as particles.

(a) Find the acceleration of the car and trailer.

(b) Find the magnitude of the tension in the tow bar.

(c) How have you used the assumption that the car and trailer can be modelled as particles?

The car is moving along the road when the driver sees a hazard ahead. He reduces the force produced by the engine to zero and applies the brakes. The brakes produce a force on the car of magnitude F N and both the car and trailer decelerate.

(d) Given that the resistances to motion are unchanged and that the magnitude of the thrust in the tow bar is 125 N, find the value of F.

R4.12 Two particles, A and B, are such that A is heavier and has a mass of 2.8 kg. The particles are connected by a light inextensible string which passes over a smooth, fixed pulley. Initially, particle A is 3.75 m above a horizontal surface. The particles are released from rest with the string taut and the hanging parts of the string vertical. After particle A has been descending for 2.5 s, it strikes the ground. Particle A reaches the ground before particle B has reached the pulley.

(a) Show that the acceleration of A as it descends is 1.2 m s^{-2}.

(b) Show that the mass of particle B is 2.2 kg.

(c) State how you have used the information that the string is inextensible.

When particle A strikes the ground, it does not rebound and the string becomes slack. Particle B then moves freely under gravity, without reaching the pulley, until the string becomes taut again.

(d) Find the time between the instant when particle A strikes the ground and the instant when the string becomes taut again.

R4.13 A parcel C, of mass 2 kg, is at rest on a smooth slope inclined at an angle θ to the horizontal, where $\tan\theta = 0.75$. C is connected to another parcel D, of mass 3 kg, by means of a light inextensible rope and a smooth pulley at the top of the slope. D is suspended above the ground.

(a) How are C and D modelled? Explain your answer.

(b) Find the initial acceleration and tension.

After two seconds the rope snaps.

(c) How long does it take C to return to its original position after the rope snaps?

R4.14 A particle U slides 32 m down a rough plane inclined at an angle θ to the horizontal, where $\cos\theta = 24/25$, pulling particle V to which it is connected by a light string. Particle U has a mass of M kg. The masses of U and V are in the ratio 5 : 2. Due to the relative sizes of the particles, the string makes an angle of 30° with the plane. The particles accelerate at $(43/625)g$ m s^{-2}. The coefficient of friction between U and the plane is 0.2.

(a) Find the coefficient of friction between V and the plane.

(b) Show that the tension in the string is given by $(8\sqrt{3} / 625)Mg$.

R4.15 Particle A is suspended from a vertical string and particle B is suspended by a vertical string from particle A. Initially, the system is at rest with particle B 2h m above the ground. The mass of particle B is 80% of the mass of particle A. A force is applied vertically upwards to A with a magnitude three times greater than the weight of B. After being pulled h m, the string between A and B breaks. Show that the velocity of B when it returns to the ground is given by $v^2 = 20gh / 3$.

R4.16 Blocks P and Q are connected by a light rope passing over a smooth pulley. Each block is at rest on either side of a wedge. P has mass M kg and sits on the smooth side. Q is 5 kg heavier than P and sits on the rough side. The coefficient of friction between Q and the wedge is 0.2. The wedge is an isosceles triangle with a right angle at the top (each slope is inclined at 45°).

(a) Given that P accelerates at $2\sqrt{2}$ m s^{-2}, find the value of M.

(b) Find the force exerted on the pulley by the rope.

R4.17 A block of mass 6.5 kg is connected to a ball of mass M kg by means of a light cord running across a smooth pulley. The block is at rest on a rough plane inclined at angle θ to the horizontal, where sin θ = 5/13. The coefficient of friction between the block and the plane is 1/3. The ball is suspended below the pulley. A force of 7g N is applied to the block down the plane along the line of greatest slope.

(a) Given that the block moves down the slope, find the maximum value of M.

(b) If the ball is 3 kg lighter than the block, find the time taken for them to move 80 cm.

R4.18 Particles P and Q, of masses 2.1 kg and 1.5 kg, respectively, are attached to the ends of a light inextensible taut string which passes over a smooth pulley fixed at the edge of a rough horizontal table. Initially, P is on the table 2.5 m from the pulley and Q hangs freely below the pulley, 1 m above the ground. The coefficient of friction between P and the table is given by μ. The system is released from rest.

(a) Show that the initial acceleration of P is given by $(5 - 7\mu)g / 12$ m s^{-2}.

After Q hits the ground, P continues to move on the table, coming to rest before reaching the pulley.

(b) Show that $1/5 < \mu < 5/7$.

R4.19 A car is towing a milk float up a hill inclined at angle β to the horizontal. The masses of the car and milk float are 400 kg and 600 kg, respectively. The engine of the car exerts a driving force of 3.2 kN. Both vehicles experience a frictional force of 100g N and accelerate at 0.2 m s^{-2}. Find:

(a) the size of β in degrees, correct to 1 decimal place

(b) the coefficient of friction between the car and the road.

Check yourself

C4.1 A crate of mass 12 kg is pulled across a rough horizontal floor by a force of 60 N inclined at 30° above the horizontal. Given that the coefficient of friction between the crate and the floor is 0.35, find the acceleration of the crate.

C4.2 A parachutist of mass 75 kg drops vertically under gravity. At an altitude of 800 m, the parachutist experiences air resistance of magnitude 540 N. Taking g = 10 m s^{-2}, calculate the downward acceleration at this moment.

C4.3 A parcel of mass 6 kg rests on a rough plane inclined at 25° to the horizontal. The parcel is on the point of slipping down the plane. Calculate the coefficient of friction μ.

C4.4 A car of mass 1100 kg tows a trailer of mass 700 kg along a level road. The car's engine provides a driving force of 2700 N. The resistive forces on the car and trailer are 350 N and 250 N, respectively. Calculate the acceleration of the combination and the tension in the tow-bar.

C4.5 Two masses of 5 kg and 3 kg are connected by a light inextensible string passing over a smooth fixed pulley. The system is released from rest.

(a) Find the acceleration of the masses and the tension in the string.

(b) Find the magnitude of the resultant force exerted on the pulley.

Chapter quiz

Q1 A resultant force of 36 N acts on a mass of 4.5 kg. What is the magnitude of the acceleration produced?

A	B	C	D
0.125 m s^{-2}	8.0 m s^{-2}	162 m s^{-2}	40.5 m s^{-2}

Q2 An astronaut has a weight of 700 N on Earth (g = 10 m s^{-2}). What is their weight on Mars where g = 3.75 m s^{-2}?

A	B	C	D
262.5 N	70 N	186.7 N	2625 N

Q3 A 15 kg box is pulled horizontally by 65 N across a rough floor (μ = 0.3, g = 10). What is its acceleration?

A	B	C	D
1.33 m s^{-2}	4.33 m s^{-2}	0.75 m s^{-2}	3.00 m s^{-2}

Q4 A 2 kg ball falls vertically under gravity (g = 10) with 4 N air resistance. What is its downward acceleration?

A	B	C	D
10 m s^{-2}	8 m s^{-2}	12 m s^{-2}	6 m s^{-2}

Q5 A 60 kg person stands in a lift accelerating upwards at 1.5 m s^{-2} (g = 10). What is the normal reaction on the person?

A	B	C	D
600 N	510 N	690 N	750 N

Q6 A lift of mass 300 kg carrying a 100 kg crate decelerates upwards at 2 m s^{-2} (g = 10). Find the cable tension.

A	B	C	D
4800 N	3200 N	4000 N	2400 N

Q7 A 5 kg block slides on a smooth slope inclined at 30° to the horizontal (g = 10). Find its acceleration down the slope.

A	B	C	D
5.0 m s^{-2}	8.66 m s^{-2}	10.0 m s^{-2}	2.5 m s^{-2}

Q8 A mass on a rough slope inclined at 30° is on the point of slipping downwards. What is the coefficient of friction μ?

A	B	C	D
0.500	0.577	0.866	1.732

Q9 A car of 800 kg tows a 400 kg trailer. A driving force of 1800 N acts, with no resistances. Find the tow-bar tension.

A	B	C	D
600 N	1200 N	900 N	1800 N

Q10 Two masses of 3 kg and 2 kg are connected over a smooth pulley (g = 10). What is the acceleration of the system?

A	B	C	D
1.0 m s^{-2}	2.0 m s^{-2}	5.0 m s^{-2}	9.8 m s^{-2}

Q11 In an Atwood machine with masses 3 kg and 2 kg (g = 10, a = 2 m s^{-2}), what is the tension in the connecting string?

A	B	C	D
20 N	24 N	25 N	30 N

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/m1-4-newtons-laws-of-motion/