M1·3

Momentum

CHAPTER ASSIGNMENT

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. State appropriate units and degrees of accuracy. Give non-exact values correct to 3 significant figures unless stated otherwise.

Practice questions from *Cambridge International AS & A Level Mathematics: Mechanics Student's Book* (Collins, Tom Andrews and Michael Kent), Chapter 3 (Momentum), Exercises 3.1A, 3.2A, 3.3A, and Exam-style questions, for classroom use. Where a question needs a diagram, data or descriptions are provided. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 3.1A · Calculating momentum

3.1.1 Calculate the momentum of:

(a) a tennis ball of mass 0.056 kg and velocity 35 m s^{-1}

(b) a piano of mass 160 kg falling with a speed of 10 m s^{-1}

(c) a lion of mass 180 kg running at a speed of 30 km h^{-1}

(d) a wrecking ball of mass 860 kg swung at 5 m s^{-1}

(e) a man of mass 80 kg running at a pace of 10 km per hour.

3.1.2 Calculate the momentum of an electron of mass 9.1×10^{-31} kg that is moving at 2.2×10^{6} m s^{-1}.

3.1.3 What is the speed of a train that has a momentum of 500 000 kg m s^{-1} and a mass of 100 metric tonnes?

3.1.4 A car of mass 1.3 metric tonnes is reversing down a driveway with a speed of 3 m s^{-1}. Calculate the momentum of the car, making the direction clear.

3.1.5 A bullet leaves the end of a gun at 860 m s^{-1} with a momentum of 38.7 kg m s^{-1}. What is the mass in grams of the bullet?

3.1.6 A boat has a mass of 120 000 kg and is travelling with a velocity of 3 m s^{-1} due north. Work out the momentum, p kg m s^{-1}, of the boat.

3.1.7 Estimate the momentum, p kg m s^{-1}, of:

(a) a cricket ball after being struck by a bat

(b) a snowflake falling from a cloud

(c) a small car on an expressway

(d) a person running the 100 m race at the Olympics.

3.1.8 A sledge sliding along a frozen lake has momentum of p kg m s^{-1}. Write an expression for the new momentum, in terms of p under the following changes:

(a) Half the mass and the same speed.

(b) Twice the mass and the same speed.

(c) Twice the mass and half the speed.

(d) A third of the mass and twice the speed.

(e) Four times the mass and half the speed.

3.1.9 An object of constant mass 8 kg is doubling in velocity every second. If it has an initial velocity of 5 m s^{-1}, after how many seconds is the momentum over 1000 kg m s^{-1}?

3.1.10 A car of mass 1.3 tonnes accelerates from a speed of 30 km h^{-1} at a rate of 3 m s^{-2} for 5 seconds.

(a) What is the final momentum of the car?

Given that the car has an initial speed of u m s^{-1}:

(b) write an expression for the final momentum of the car.

3.1.11 Write an expression for the change in momentum of an object of mass m kilograms that has been uniformly accelerating for t seconds for a distance of s metres.

Exercise 3.2A · Conservation of linear momentum

3.2.1 For each of these direct impacts, calculate the missing mass or velocity (taking the positive direction to the right):

(a) Before: 3 kg moving right at 12 m s^{-1}, 2.8 kg moving right at 2 m s^{-1}. After: 3 kg moving right at 5 m s^{-1}, 2.8 kg moving right at u m s^{-1}.

(b) Before: 2 kg moving right at 15 m s^{-1}, 10 kg moving right at 12 m s^{-1}. After: 2 kg moving right at u m s^{-1}, 10 kg moving right at 16 m s^{-1}.

(c) Before: m kg moving right at 12 m s^{-1}, 0.1 kg moving right at 3 m s^{-1}. After: m kg moving right at 7 m s^{-1}, 0.1 kg moving right at 13 m s^{-1}.

(d) Before: 3 kg moving right at 4 m s^{-1}, 5 kg moving left at 4.4 m s^{-1}. After: 3 kg moving left at 12.5 m s^{-1}, 5 kg moving right at u m s^{-1}.

3.2.2 Teertha and her younger brother Kian are at the ice rink. They stand facing each other, then Teertha pushes against Kian and they both move backwards. Teertha estimates that she is moving at 1.5 m s^{-1}. Given that Teertha has mass 40 kg and Kian has mass 30 kg, how fast is Kian moving?

3.2.3 In a theme park ride, a bumper car of mass 150 kg is travelling at 3 m s^{-1}. It hits a stationary bumper car of mass 120 kg head on, and the second bumper car then starts to move at 2 m s^{-1}. What is the subsequent velocity of the first car?

3.2.4 Two inflatable boats collide head on. The total mass (including the passengers) of boat A is 120 kg, and the total mass of boat B is 100 kg. Boat B was travelling toward boat A at 1.8 m s^{-1}, and boat A was travelling at 1.5 m s^{-1} in the opposite direction. After the collision, boat A bounces backwards with a speed of 1.2 m s^{-1}. What are the new speed and direction of boat B after the collision?

3.2.5 A snooker player hits the white ball, causing it to move at 0.8 m s^{-1}. The white ball directly hits a red ball, which has the same mass and size as the white. The red ball moves forward at 1.3 m s^{-1} in the same line as the white ball. In which direction does the white ball move and at what speed?

3.2.6 When a naval cannon is fired it will move backwards, so it has a thick piece of rope which is designed to slow it down after firing. If a cannon has a mass of 100 kg, and is loaded with a cannonball of mass 5.5 kg, what is the initial speed of the cannon after the cannonball is fired at a horizontal speed of 140 m s^{-1}?

3.2.7 Two cars crash on a smooth, straight road. The report states that the cars are travelling at 5 m s^{-1} and 4 m s^{-1}. It also notes that the cars have masses of 1200 kg and 1000 kg. The report does not state which car is which, or the initial directions of the cars. The cars collided, but did not stick together. How many different solutions are there for the subsequent movement of the two cars? The 1000 kg car was brought to a stop by the crash. Work out the new speed of the 1200 kg car in each case, making the direction clear.

3.2.8 Three snooker balls, A, B and C, of equal mass are spaced apart in a straight line. B and C are initially at rest. Ball A is hit such that it now has an initial velocity of u m s^{-1}, moving towards B. After hitting B, A now moves with a speed of v_1. B moves at a speed of v_2 and it collides with C causing it to rebound with a speed of v_3. Ball C now moves at a speed of v_4. Given that ball B does not meet ball A for a second time, write an inequality for u in terms of the final velocities of balls A and C.

3.2.9 A light football of mass 200 g, initially at rest, starts rolling down a 3 m long slope. It accelerates at a rate of 6 m s^{-2}. At the bottom of the hill it directly hits a standard football of mass 400 g, causing the light football to stop moving. How fast is the standard football moving?

3.2.10 Isabel and Gabriel are standing together on an ice rink. They push against each other and move such that it takes Gabriel 4 seconds to move 10 m. How long does it take Isabel to move 10 m:

(a) if Isabel and Gabriel weigh the same amount

(b) if Isabel weighs twice as much as Gabriel?

3.2.11 A ball of mass 100 g is projected along a straight line with a velocity of 3 m s^{-1}. After 5 metres it meets another ball of equal mass. Given that it takes 2 seconds to return to its initial position, find the velocity of both balls after they collide, stating the direction of motion clearly.

Exercise 3.3A · Coalescing bodies

3.3.1 These diagrams show two bodies which coalesce on impact. Calculate the missing mass or velocity in each case (taking the positive direction to the right):

(a) Before: 6.8 kg moving right at 11 m s^{-1}, 6.7 kg moving right at 2 m s^{-1}. After: combined mass 13.5 kg moving right at v m s^{-1}.

(b) Before: 20 kg moving right at 5 m s^{-1}, 80 kg moving right at u m s^{-1}. After: combined mass 100 kg moving right at 1.8 m s^{-1}.

(c) Before: 15 kg moving right at 5 m s^{-1}, m kg moving left at 9 m s^{-1}. After: combined mass moving left at 6 m s^{-1}.

(d) Before: 120 g moving right at 0.1 m s^{-1}, 1.38 kg moving right at u m s^{-1}. After: combined mass moving right at 0.031 m s^{-1}.

3.3.2 A man throws a large bag of rice of mass 10 kg onto a shopping cart of mass 35 kg. Given that the bag lands horizontally at 2 m s^{-1}, find the speed at which the trolley starts to move.

3.3.3 An empty truck is coasting along a frictionless horizontal track. It starts to rain. State whether, as the rain collects in it, the truck will speed up, slow down or remain at the same speed, giving justification for your answer.

3.3.4 Lynda has a pet dog, Scruff, which has a mass of 2 kg. They are standing in the park when Scruff spots a cat. Scruff jumps up to chase after it. Just as his leash goes taut, Scruff is travelling at a velocity of 13 m s^{-1}. Find the common speed of Scruff and Lynda, given that Lynda has a mass of 63 kg.

3.3.5 A bullet of mass 0.01 kg is fired into a block of wood of mass 2 kg at a speed of 540 m s^{-1}. The bullet is embedded into the block. Find the speed of the block immediately after the impact.

3.3.6 A particle of mass 4m kg and velocity 2u m s^{-1} strikes a particle of mass 2m kg. The two particles coalesce. What is the velocity of the combined particle, as a multiple of u, given that the 2m particle has the following initial conditions?

(a) It is initially at rest.

(b) Initially, the 2m particle is moving at u m s^{-1}.

(c) What would happen if, initially, the 2m particle is moving at 3u m s^{-1}?

3.3.7 Given that two objects, A and B, with masses m_A and m_B and initial velocities u_A and u_B, coalesce and move with a common velocity, v, write an expression for m_A in terms of m_B, u_A, u_B and v.

3.3.8 A ball of mass 1 kg moving in a tube with a velocity of 14 m s^{-1} experiences a small internal explosion that causes it to break into two pieces moving in opposite directions. One piece has mass 400 g and velocity 47 m s^{-1} in the same direction as the ball was travelling. Find the immediate velocity of the second piece of the ball.

3.3.9 Sebastian makes a cart, which he can jump onto and move. He decides to test it on a smooth, flat road. He stands on it, but it does not move. He tries again, jumping from a standing position and landing on the cart with a horizontal speed of 0.5 m s^{-1}. Sebastian has a mass of 20 kg, and the cart has a mass of 12 kg.

(a) Calculate the speed of the cart with Sebastian riding on it.

He tries a running start, starting 2 m away from the cart and accelerating from rest at a rate of 0.25 m s^{-2}.

(b) Calculate the new speed of the cart with Sebastian riding on it.

He wants the cart to go at a speed of 2 m s^{-1}.

(c) How far will he have to run at this same rate of acceleration, before jumping into the cart, to achieve this final speed? Is this a realistic situation?

3.3.10 Two balls collide directly: ball A of mass m_A kg has initial velocity u m s^{-1} and ball B of mass m_B kg is initially at rest. After impact, ball A moves with velocity v_A m s^{-1} and ball B moves with velocity v_B m s^{-1} in the positive direction.

(a) Write an expression for v_B in terms of u, v_A, m_A and m_B.

(b) Given that A continues to travel towards B, and A has half the mass of B, write an inequality for u in terms of v_B.

Exam-style questions

R3.1 Calculate the momentum of a ball of mass 0.5 kg and a velocity of 4.5 m s^{-1}. It collides with a block of mass 3 kg that is free to move. The block is initially stationary, but upon being hit it moves at v m s^{-1}. For which values of v does the ball rebound, changing direction?

R3.2 An object has a momentum of 54 kg m s^{-1}.

(a) Given that it is travelling at 35 km h^{-1} calculate the mass of the object.

(b) The object loses half of its mass, but its total momentum is conserved. Write down the new velocity of the object.

R3.3 A car safety testing centre wishes to simulate the effect of a car of mass 1500 kg crashing head-on into a car of mass 1300 kg. Both cars are to be travelling at 36 km h^{-1}.

(a) What is the momentum of each car? Give your answer in kg m s^{-1}.

(b) The test centre only has the facilities to have one moving car, but can crash it into a wall. It chooses the 1500 kg car, and accelerates it to v m s^{-1} just as it hits the wall. Given that it wishes to make the total magnitude of the momentum the same as if the two cars were meeting, calculate the value of v m s^{-1}.

R3.4 Particle A, of mass 2m and initial velocity 3u, collides with particle B, of mass 3m and initial velocity 2u. Calculate the final velocity of:

(a) A if B has a final velocity of 4u

(b) A if B has a final velocity of 3u

(c) both particles, given that they combine

(d) B if A has a final velocity of −3u.

R3.5 Three balls A, B and C of masses 1 kg, 2 kg and 3 kg are in a line on a smooth horizontal surface with initial velocities 12 m s^{-1}, 0 m s^{-1} and 0 m s^{-1}, such that ball A is travelling towards ball B.

(a) What is the initial momentum of ball A?

(b) When A collides with B, ball A stops moving. What is the velocity of B when it is in motion?

(c) Subsequently, when B collides with C, ball B stops moving. What is the final velocity of C?

(d) Write down the final momentum of ball C, explaining the significance of your answer.

R3.6 Two identical beads are free to move along a smooth horizontal bar such that any collisions are direct. Initially, the beads are at rest 2 m apart. A force is applied to A for 3 seconds such that it accelerates at a rate of 0.4 m s^{-2} towards bead B.

(a) Show that when the force is removed bead A has not reached bead B.

(b) Bead A subsequently collides with bead B. Given that bead A stops moving after the collision, find the velocity of B.

R3.7 Basketball and golf ball collision and gun recoil:

(a) A basketball weighs 13 times more than a golf ball. If they are thrown at each other with an equal and opposite velocity, u, show that $v_g = 12u - 13v_b$, where v_g is the velocity of the golf ball after impact and v_b is the velocity of the basketball after impact.

(b) A bullet of mass m kg is fired from a gun of mass M kg, which is free to recoil. Let the initial velocities of the bullet and gun be u m s^{-1} and U m s^{-1}, respectively. Show that the initial speeds of the bullet and gun are in inverse ratio to their masses.

R3.8 An experiment is set up such that five trucks, each of mass 45 000 kg, are lined up along a smooth horizontal railway line. Initially the trucks are all stationary. The first truck, A, is then speeded up to a velocity of 12 m s^{-1}.

(a) Truck A meets truck B and the two join together. Calculate the velocity of the two trucks.

(b) The trucks continue to roll freely along the tracks, such that A and B meet C, then A, B and C meet D, and finally all five trucks travel together. Show that the final velocity of the five trucks is 2.4 m s^{-1}.

R3.9 Two objects A and B, of mass 4.5 kg and m kg approach each other at speeds of 4 m s^{-1} and 6 m s^{-1}, respectively. They coalesce on impact.

(a) Calculate the value of m if the two objects cease to move after impact.

(b) Calculate the value of m if they move at 2.4 m s^{-1} in the direction of B after impact.

(c) In fact, m = 18 kg. Calculate the final velocity of the combined mass, making the direction clear.

(d) After a short time, the combined mass breaks apart into two sections with masses 10.5 kg and 12 kg. The 10.5 kg section moves in the same direction as the combined mass but with a speed of 1 m s^{-1}. Calculate the velocity of the second mass.

R3.10 Two discs, X and Y, of mass m and 2m, respectively, are sliding along a smooth horizontal surface. They collide directly. Prior to the collision, the velocity of X is $4u$ m s^{-1} and the velocity of Y is u m s^{-1}. Let the velocity of X after the collision be v_x m s^{-1} and the velocity of Y after the collision be v_y m s^{-1}, with $u > 0$.

(a) Show that $v_x = 6u - 2v_y$.

(b) Given that Y is now moving 150% faster than before the collision, work out the percentage decrease of the speed of X.

R3.11 Noel makes a Newton's cradle which has five balls made from different materials, so that they are the same shape but different masses. They have masses in the ratio 5:4:3:2:1 from left to right. He lifts up the heaviest one and it swings such that the velocity upon hitting the next-heaviest ball is 10 cm per second.

(a) Assuming that the heaviest ball then stops moving (as in a normal Newton's cradle), how fast will the lightest ball, at the opposite end, start to move?

(b) Noel changes the position of the balls to the order 5:3:2:4:1 and lifts the first ball in the same way. How fast does the lightest ball, at the opposite end, start to move in this situation? Justify your answer fully.

(c) Finally, he changes the position of the balls to the order 3:5:1:2:4 and lifts the first ball (now the middle-sized weight) in the same way. How fast does the last ball, at the opposite end, start to move in this situation?

R3.12 A car of mass 2000 kg has broken down on a level crossing, 125 m from the end of a station platform, which is 160 m long. Everybody gets out the car, but the approaching train, from the other side of the station, cannot be contacted. It is a goods train of mass 200 000 kg which did not need to stop at the platform. Before reaching the platform, the train driver notices the car and applies the brake. A witness notices that it takes the front of the train 5 seconds to pass the platform and a further 5 seconds to reach the car. Calculate the momentum of the train as it hits the car and the speed of the combined car and train assuming that they coalesce.

R3.13 A bus of mass 10 tonnes is following a car of mass 1.2 tonnes at a distance of 30 m. Initially, both are travelling at a constant speed of 36 km h^{-1}. The driver's foot slips and causes the bus to accelerate uniformly. After 5 seconds, the bus crashes into the back of the car, and the two vehicles coalesce. Find the acceleration (in m s^{-2}) of the bus and the velocity (in km h^{-1}) of the vehicles immediately after the crash.

R3.14 Ball A (of mass 2 kg) and ball B (of mass 3 kg) collide and coalesce. Their final velocity, after colliding, is 3 m s^{-1}. Ten seconds before they coalesced, both balls started from rest and they were accelerating at different rates. Given that ball B accelerated uniformly for the ten seconds at 0.1 m s^{-2}, find the uniform acceleration of ball A. How far apart were they ten seconds before they collided?

R3.15 A railway engine of mass 25 000 kg is positioned 50 m behind two carriages, each of mass 15 000 kg, which are also 50 m apart. The resistances to forward motion are 2 kN on the engine and 0.5 kN on each of the carriages. The train is on a straight flat section of the track. The engine begins at rest, and starts to produce a driving force of 18 kN. As the engine meets the carriages, they instantaneously join together to form a larger train. Calculate the final velocity of the engine after it has connected with the second carriage.

Mathematics in life and work · Multi-stage separation

M3.1 A space agency is designing a new way to increase the speed of deep-space probes by repeatedly splitting the shuttle into smaller sections. Initially, a shuttle of mass 160 000 kg has a velocity of u km s^{-1}. The shuttle then splits into two sections of equal mass such that one of the sections is now moving at a velocity of u km s^{-1} in the opposite direction. Show that the velocity of the second section of the shuttle is now 3u km s^{-1}.

M3.2 This second piece then splits into two smaller sections of equal mass. Again the rear section has the same speed, but the opposite direction. This continues until the mass of the shuttle is 20 000 kg. Calculate the velocity of the shuttle at each step and hence find the final velocity of the shuttle of mass 20 000 kg as a multiple of u.

M3.3 An alternative approach to speeding up a shuttle would be to eject the smallest shuttle, of mass 20 000 kg, directly from the largest shuttle, of mass 160 000 kg. The final shuttle must attain the same speed by either method. Given that u = 140 km s^{-1}, what is the velocity of the two sections after the first, and only, separation using this second approach?

Check yourself

C3.1 A tennis ball of mass 0.06 kg is dropped vertically from rest from a height of 1.8 m onto a horizontal concrete floor. It rebounds to a height of 1.25 m. Taking g = 9.8 m s^{-2}:

(a) Find the speed of the ball immediately before impact.

(b) Find the speed of the ball immediately after impact.

(c) Taking upwards as positive, calculate the impulse exerted by the floor on the ball.

(d) If the impact lasts 0.015 seconds, calculate the average force exerted by the floor on the ball.

C3.2 A truck of mass 3500 kg travelling at 18 m s^{-1} collides head-on with a car of mass 1500 kg travelling at 22 m s^{-1} in the opposite direction. The vehicles lock together upon impact (coalesce).

(a) Calculate the common velocity of the wreckage immediately after the impact, stating its direction clearly.

(b) Calculate the total kinetic energy of the two vehicles before the impact.

(c) Calculate the kinetic energy immediately after the impact, and determine the mechanical energy dissipated.

C3.3 Two smooth trolleys P and Q of masses 2 kg and 5 kg respectively are moving towards each other on a smooth horizontal track. Trolley P has velocity 8 m s^{-1} to the right and trolley Q has velocity 4 m s^{-1} to the left. After colliding, trolley P rebounds to the left with speed 2 m s^{-1}.

(a) Find the magnitude and direction of the velocity of trolley Q after the collision.

(b) Calculate the magnitude of the impulse exerted by trolley P on trolley Q during the collision.

C3.4 A projectile of mass 12 kg is travelling horizontally at 240 m s^{-1} when it explodes into two fragments of mass 4 kg and 8 kg. Immediately after the explosion, the 4 kg fragment moves in the opposite direction at 60 m s^{-1}. Find the velocity of the 8 kg fragment immediately after the explosion.

C3.5 A pile-driver of mass 600 kg falls vertically from rest through a distance of 2.5 m under gravity before striking a stationary vertical pile of mass 1400 kg. The driver remains in contact with the pile without rebounding. Taking g = 9.8 m s^{-2}:

(a) Calculate the speed of the driver immediately before striking the pile.

(b) Calculate the common speed of the pile and driver immediately after the impact.

(c) The pile is driven 8 cm into the ground before coming to rest. Find the average resistance force of the ground.

Chapter quiz

Q1 What is the linear momentum of a 0.25 kg ball moving with a velocity of 16 m s^{-1}?

A	B	C	D
4 kg m s^{-1}	0.0156 kg m s^{-1}	64 kg m s^{-1}	2 kg m s^{-1}

Q2 Which SI unit is equivalent to the unit of momentum, kg m s^{-1}?

A	B	C	D
N m	N s	J s	W s

Q3 Two trolleys collide on a smooth horizontal floor. When is their total linear momentum conserved?

A	B	C	D
Only if the collision is perfectly elastic with no energy loss	Only if the two trolleys coalesce and move together after impact	Whenever no external resultant horizontal force acts on the system	Only if both colliding bodies have equal mass and opposite speeds

Q4 A 2 kg mass at 6 m s^{-1} collides with a stationary 4 kg mass and coalesces. What is their common speed?

A	B	C	D
3 m s^{-1}	2 m s^{-1}	1 m s^{-1}	4 m s^{-1}

Q5 A 1200 kg car travelling at 20 m s^{-1} is brought to rest in 4 s. What is the magnitude of the average braking force?

A	B	C	D
4800 N	3000 N	6000 N	12 000 N

Q6 A 0.05 kg bullet is fired horizontally at 400 m s^{-1} from a 2.5 kg rifle. What is the rifle recoil speed?

A	B	C	D
4 m s^{-1}	16 m s^{-1}	2 m s^{-1}	8 m s^{-1}

Q7 Ball A (3 kg at 4 m s^{-1}) hits stationary Ball B (1 kg). If A continues at 2 m s^{-1}, find the speed of B.

A	B	C	D
6 m s^{-1}	2 m s^{-1}	4 m s^{-1}	8 m s^{-1}

Q8 A 0.2 kg ball hits a wall horizontally at 15 m s^{-1} and rebounds at 10 m s^{-1}. What is the impulse magnitude?

A	B	C	D
1 N s	5 N s	2.5 N s	0.5 N s

Q9 In an inelastic collision between two free particles, which statement is always true?

A	B	C	D
Both total momentum and total kinetic energy are conserved	Total momentum is conserved, but mechanical kinetic energy is lost	Total kinetic energy is conserved, but total momentum changes	Both momentum and kinetic energy decrease to zero instantaneously

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Q10 What physical quantity is represented by the area under a force–time (F–t) graph?

A	B	C	D
Work done by the force	Rate of change of kinetic energy	Acceleration of the mass	Impulse (change in linear momentum)

Q11 Two ice skaters push apart from rest. Skater A is 60 kg at 2 m s^{-1}. What is the speed of Skater B (40 kg)?

A	B	C	D
1.33 m s^{-1}	2.5 m s^{-1}	3 m s^{-1}	4 m s^{-1}

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/m1-3-momentum/