M1·1

Forces and equilibrium

CHAPTER ASSIGNMENT

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given, showing all your working. State appropriate units (N, kg, degrees) and degrees of accuracy. Take the acceleration due to gravity g = 10 m s^{-2} where applicable. Give non-exact values correct to 3 significant figures unless stated otherwise.

Practice questions from *Cambridge International AS & A Level Mathematics: Mechanics Student's Book* (Collins, Tom Andrews and Michael Kent), Chapter 1 (Forces and equilibrium), Exercises 1.1A, 1.2A, 1.2B, 1.3A, 1.4A, and Key points and Exam-style questions (Chapter review 1), for classroom use. Where a question needs a diagram, the diagram is described in words or geometric parameters are provided. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 1.1A · Forces and equilibrium

1.1A.1 Draw diagrams for the following.

(a) A particle is at rest on a horizontal surface.

(b) A suitcase is pulled across a rough horizontal table by a horizontal rope.

(c) A particle is pushed across a smooth horizontal table.

1.1A.2 A particle experiences four horizontal forces, A, B, C and D, as shown in the diagram (forces A to the left, C to the right, B upwards, D downwards). If the particle is stationary, what can you say about the forces?

1.1A.3 A toy train is pulled along a horizontal table at a steady speed by a taut string. The frictional force is given by X. The weight of the train is 4 N. Label the missing forces in the diagram (normal reaction upwards, weight downwards, string tension forward, friction X backward).

1.1A.4 A box is pushed by a thrust force at a steady speed along a rough horizontal table. The frictional force is 12 N. The reaction force exerted on the box by the table is 58 N.

(a) Draw a force diagram to represent the four forces acting upon the box.

(b) Calculate the magnitude of the thrust force.

(c) Calculate the weight of the box.

Exercise 1.2A · Resolving forces in equilibrium

1.2A.1 Draw diagrams for the following.

(a) A particle is at rest on a rough plane inclined at θ to the horizontal.

(b) A particle is pulled across a rough horizontal table by a string inclined at 60° to the horizontal.

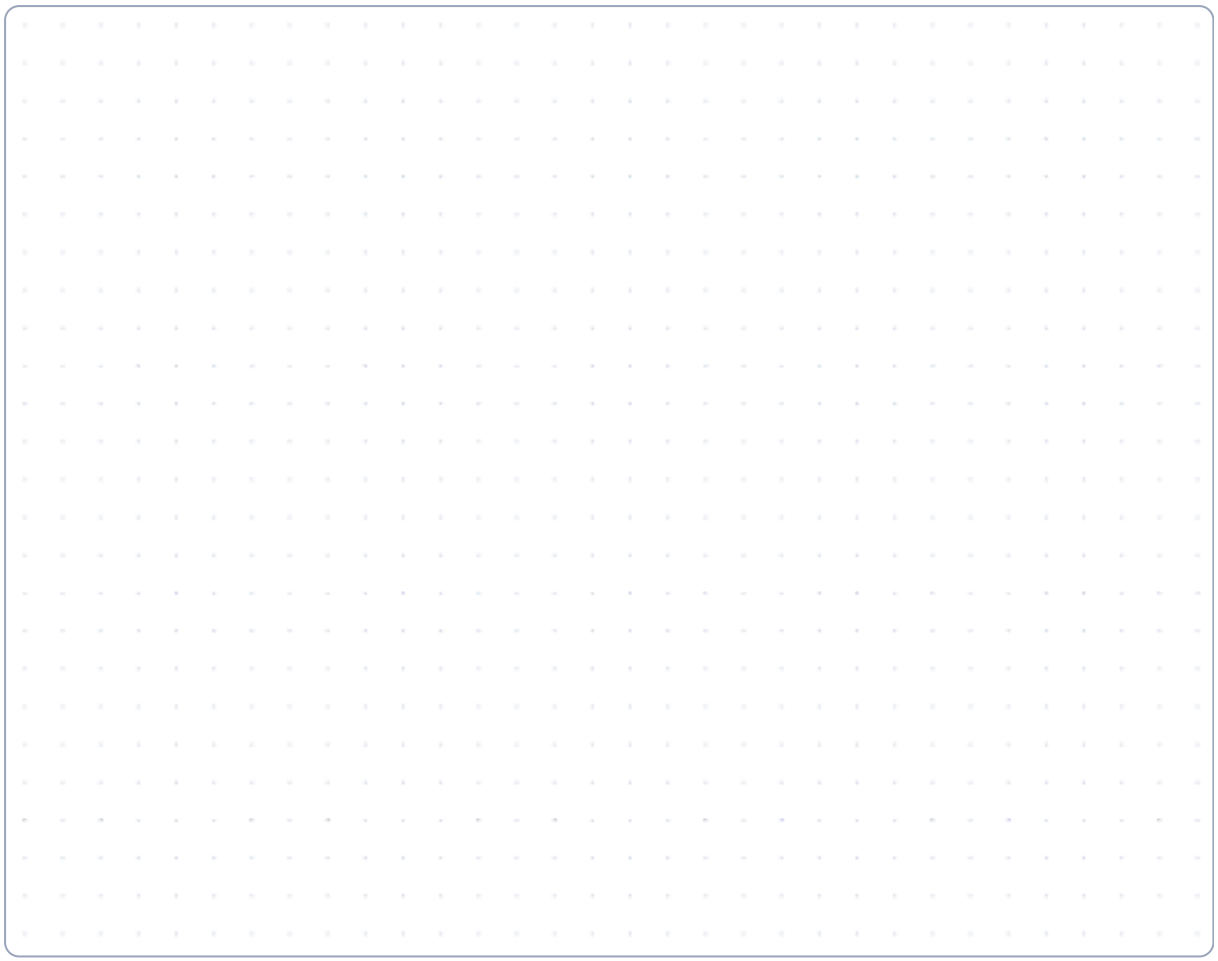
(c) A particle is held at rest on a smooth plane inclined at 30° to the horizontal by a string parallel to the plane.

(d) A car drives up a rough slope inclined at θ to the horizontal.

(e) A particle is on the verge of moving up a rough slope (inclined at 26° to the horizontal) as a result of being pushed by a horizontal force P.

(f) A particle is just held in position on a rough slope (inclined at 26° to the horizontal) by a horizontal force P.

(g) A particle is suspended from a horizontal beam by two unequal strings at angles of 50° and 20° to the beam.



1.2A.2 A peg bag of weight W N is suspended from a string such that both ends of the string are inclined at θ to the horizontal and the tensions in both ends are identical and given by T N, as shown in the diagram. The system is in equilibrium.

(a) Resolve vertically.

(b) Hence find an expression for T in terms of W and θ.

1.2A.3 A toy bus of weight W N is pulled along a rough horizontal carpet at a constant speed by a string. The string makes an angle of 19° with the carpet, as shown in the diagram. The system is in equilibrium.

(a) Resolve parallel to the carpet.

(b) Resolve perpendicular to the carpet.

1.2A.4 A particle of weight W N is suspended from two strings, A and B, as shown in the diagram. String A is at an angle of 25° to the horizontal and string B is at an angle of 65° to the horizontal. The system is in equilibrium.

(a) Resolve horizontally.

(b) Resolve vertically.

1.2A.5 Each diagram shows a system in equilibrium. For each diagram, Mary has been asked to resolve in a given direction. Explain what is wrong with Mary's answer in each case and give the correct answer.

(a) Resolve perpendicular to the slope (slope angle 24°, horizontal force X rightwards, weight W downwards, normal reaction R perpendicular up, friction F up slope). Mary's answer: R = W cos 24°.

(b) Resolve parallel to the slope (slope angle 24°, force X parallel up slope, friction F up slope, weight W downwards). Mary's answer: F + X cos 24° = W sin 24°.

(c) Resolve parallel to the slope (slope angle 24°, horizontal force X rightwards, friction F down slope, weight W downwards). Mary's answer: X cos 24° = F + W cos 24°.

(d) Resolve perpendicular to the slope (slope angle 24°, horizontal force X rightwards, friction F down slope, weight W downwards). Mary's answer: R = X cos 24° + W cos 24°.

(e) Resolve parallel to the slope (slope angle 24°, force X directed at 64° to the horizontal, friction F down slope, weight W downwards). Mary's answer: X cos 64° = F + W sin 24°.

1.2A.6 A clock of weight W N is at rest on a rough slope inclined at angle θ. The clock experiences a normal reaction of R N and a frictional force of F N. Show that F = R tan θ.

1.2A.7 A piano of weight W N is pushed up a rough slope at a steady speed by a horizontal force of X N. The slope is inclined at angle θ. The piano experiences a normal reaction of R N and a frictional force of F N.

(a) Resolve parallel to the slope.

(b) Resolve perpendicular to the slope.

(c) Hence show that X can be written as R sin θ + F cos θ.

1.2A.8 A freezer of weight 250 N is being pushed at a steady speed up a rough path inclined at an angle of 12° to the horizontal, as shown in the diagram. The pushing force, X N, is horizontal. Given that the magnitude of the frictional force is one-quarter the magnitude of the normal reaction force, show that X is approximately 120 N.

1.2A.9 A box of weight W N is pulled along a rough horizontal road at a steady speed by a rope inclined at 30° to the horizontal, as shown in the diagram. R and F are the normal reaction and friction forces, respectively. Show that W is given by the expression $(3R + \sqrt{3}F)/3$.

1.2A.10 A block of weight W N is pulled at a steady speed up a rough plane inclined at an angle of 27° by a cable of tension T N inclined at an angle of 15° to the plane. The reaction and friction forces are given by R N and F N, respectively. Given that F = (1/2) R, show that T/W = (cos 27° + 2 sin 27°) / (2 cos 15° + sin 15°).

Exercise 1.2B · Further resolving and equilibrium

1.2B.1 A 50 N chair is at rest on a slope inclined at 28° to the horizontal, as shown in the diagram. The chair is held in position by friction. Find the magnitude of:

(a) the reaction force

(b) the friction force.

1.2B.2 A W N particle is suspended from two identical strings. Each string is inclined at an angle of $\sin^{-1}(12/35)$ to the horizontal. Given that the tension in each string is 32 N, find the value of W.

1.2B.3 A particle of weight 38 N, suspended from a rope inclined at 20° to the vertical, is kept in position by a horizontal force X N, as shown in the diagram. Victrix and Colin have been asked to find the value of X, correct to 3 significant figures.

Victrix's solution is as follows:

R(↑) $T \cos 20° = 38 \Longrightarrow T = 38 \div \cos 20° = 40.439$ N

R(→) $X = T \sin 20° = 40.439 \times \sin 20° = 13.8$ N (3 s.f.)

Colin's solution is as follows:

R(↗) $X \cos 20° - 38 \sin 20° = 0 \Longrightarrow X = 38 \tan 20° = 13.8$ N

Compare and contrast the two solutions.

1.2B.4 A particle at rest on a horizontal table experiences three horizontal forces, P, Q and A. Forces P and Q are perpendicular. Force A acts at 30° to the line of action of Q, as shown in the diagram. Q is 4 N larger than P.

(a) Explain why the weight of the particle is not required to find any of the forces P, Q or A.

(b) Find the magnitude of A.

1.2B.5 A case of weight 30 N is dragged along a rough horizontal floor by a cord inclined at $\sin^{-1} 0.2$ to the horizontal. The case experiences a frictional force of $\sqrt{6}$ N. Find the reaction exerted by the floor on the case.

1.2B.6 A box of bricks weighs 120 N. The box is at rest on a rough plane inclined at an angle θ above the horizontal, where cos θ = 40/41. A 20 N force acts on the particle up the line of greatest slope of the plane. Find the magnitude and direction of the frictional force.

1.2B.7 The diagram shows a 15 N peg bag suspended from two unequal strings at 55° and 35° to the horizontal. Find the tension in each string.

1.2B.8 A pegbag of weight 20 N is suspended from a light cord such that one end of the cord, with a tension of T N, makes an angle of α with the horizontal and the other end, with a tension of U N, makes an angle of 2α with the horizontal, as shown in the diagram.

(a) Show that U = (20 cos α) / (cos 2α sin α + sin 2α cos α).

(b) Given that α = 30°, show that T = 10.

1.2B.9 Three horizontal forces, P, Q and R, act upon a particle, as shown in the diagram. Q and R are perpendicular. The angle between P and Q is obtuse and equal to β. Show that Q + R = P(sin β − cos β).

1.2B.10 A pulley of weight W N is suspended from a string, as shown in the diagram. One end of the string is inclined at an angle α, where cos α = 12/37, with a tension of T_1 N. The other end of the string is inclined at an angle β with a tension of T_2 N. Given that T_1 is twice the magnitude of T_2, show that $W = (T_2(70 + \sqrt{793})) / 37$.

Exercise 1.3A · Adding forces and finding resultants

1.3A.1 Forces A and B act along O_x and O_y, respectively, with magnitudes 15 N and 8 N.

(a) Find the magnitude of the resultant force.

(b) Find the angle that the resultant force makes with O_x.

1.3A.2 A particle experiences a force of 23 N south and of 38 N west.

(a) Find the magnitude of the resultant force.

(b) Find the bearing of the resultant force.

1.3A.3 Force G of magnitude 9 N acts along O_X. Force H of magnitude 7 N acts at 50° anticlockwise from O_X.

(a) Show that the resultant force is 14.5 N.

(b) Show that the resultant force acts at an angle of 21.7° to O_X.

1.3A.4 Force U of magnitude 11 N acts along O_X and force V of magnitude 7 N acts at 110° anticlockwise from O_X, as shown in the diagram. Find the magnitude and direction of the resultant force.

1.3A.5 Forces P and Q, of magnitudes 3 N and 4 N, act along O_x and O_y, respectively. Force R, of magnitude 5 N, acts at 60° to O_x as shown in the diagram. Hita has been asked to find the magnitude and direction of the resultant force.

Hita has written out this solution:

$a^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \cos 120° = 49 \Longrightarrow a = 7$

$(\sin X)/5 = (\sin 120°)/7 \Longrightarrow X = 38.2°$

$a^2 = 7^2 + 4^2 - 2 \times 7 \times 4 \cos 128.2° \Longrightarrow$ Magnitude = 9.98 N

$(\sin Z)/4 = (\sin 128.2°)/9.98 \Longrightarrow Z = 18.4° \Longrightarrow$ Angle = 56.6°

(a) Explain each part of Hita's solution.

(b) Write out a more efficient solution for this problem.

1.3A.6 Force C of 18 N acts along O_x. Force D acts at 52° to the horizontal. The magnitude of the resultant force is 28 N, as shown in the diagram.

(a) Find the magnitude of D.

(b) Find the angle that the resultant force makes with O_x.

1.3A.7 The force P acts along O_x. The forces Q, R and S act at 40°, 130° and 220°, respectively, to O_x, each measured anticlockwise from O_x. The forces P and Q both have magnitude 3 N and the forces R and S both have magnitude 5 N.

(a) Show that the resultant force acts at approximately 124° anticlockwise from O_x.

(b) Show that the magnitude of the resultant force is approximately 3 N.

1.3A.8 A toy car experiences a force of 2x N on a bearing of 000° and a second force of x N at a bearing of (180 + α)°.

(a) Show that the resultant force can be written as $x\sqrt{(5 - 2\cos\alpha)}$.

(b) Given that the resultant force is equal to 2x N, find the size of the angle α.

1.3A.9 Three horizontal forces, R, S and T, act upon a particle. Force T acts due south. Forces R and S act on bearings of (360 − θ)° and (90 − θ)°, respectively. Force R is twice the magnitude of force S.

(a) Show that $\tan\theta = 1/2$.

(b) Show that T = S √5.

1.3A.10 A boat is travelling on a bearing of 050° with a driving force of 500 N. The wind, blowing at a force of 100 N on a bearing of 160°, is taking the boat off course.

(a) Show that the new bearing is approximately 061°.

(b) Find the direction that the boat needs to take such that, after the wind has been taken into account, the boat will travel on a bearing of 050°.

Exercise 1.4A · Coefficient of friction and limiting equilibrium

1.4A.1 A garden ornament of weight 80 N sits at rest on a concrete path inclined at 28° to the horizontal, as shown in the diagram. The ornament is in limiting equilibrium. Find the coefficient of friction between the ornament and the path.

1.4A.2 A car of weight 6 kN is driven up a road at a constant speed. The road is modelled as a rough surface inclined at 8° to the horizontal. Given that the coefficient of friction between the car and the road is 0.12, find the magnitude of the driving force exerted by the engine of the car.

1.4A.3 A small suitcase of weight 50 N is pulled along the floor of an airport at a steady speed by means of a light inextensible cord inclined at 20° to the horizontal, as shown in the diagram. The coefficient of friction between the suitcase and the floor is 1/4.

(a) Show that the tension in the cord is approximately 12.2 N.

(b) How would the model change if the floor was smooth?

1.4A.4 A cyclist of weight 650 N is freewheeling at a steady speed down a hill inclined at an angle α to the horizontal on a bicycle of weight 280 N. Given that the coefficient of friction between the bicycle and the hill is 0.3, find the value of α, correct to 1 decimal place. The cyclist can be modelled as a particle.

1.4A.5 A body of weight 32 N is held in limiting equilibrium on a rough plane inclined at 25° to the horizontal by a horizontal force P. Find the magnitude of P when the body is on the point of slipping down the plane, given that $\mu = 0.15$.

1.4A.6 A piano of weight W N is being pushed up a rough slope at a constant speed. The pushing force of 560 N acts on the piano up the line of greatest slope of the plane. The coefficient of friction between the slope and the piano is 2/7 and the slope is inclined at $\sin^{-1}(1/10)$ to the horizontal. Find the weight of the piano.

1.4A.7 A car of weight 8500 N is on the point of slipping up a rough hill when a force P N is applied to the car along the line of greatest slope of the plane. The hill is inclined at an angle of 50° to the horizontal.

(a) Find the value of μ if the force P is 15 kN.

(b) Given instead that $\mu \geq 1$, find the minimum value of P.

1.4A.8 The diagram shows a metal block of mass W N being pulled up a rough slope at a steady speed by a rope with tension equal to (1/2) W N. The slope is inclined at angle θ to the horizontal and the rope is inclined at angle θ to the slope. The coefficient of friction between the metal block and the slope is μ.

(a) Show that $\mu = (1 - 2\tan\theta) / (2 - \tan\theta)$.

(b) Show that $\mu > 1/2$.

(c) Find tan θ when μ = 3/10.

1.4A.9 A skateboard of weight 30 N is on a rough plane inclined at an angle of 30° to the horizontal, held at rest by a force of P N applied parallel to the line of greatest slope of the plane. The coefficient of friction between the skateboard and the plane is 0.1.

(a) Find the minimum value for P.

(b) Find the maximum value for P.

(c) Explain why the difference between the minimum and maximum values for P is given by $3\sqrt{3}$ N.

1.4A.10 A shopping trolley is pushed up a rough slope inclined at angle α to the horizontal. When the trolley is empty it has a weight of 170 N and requires a pushing force of 40 N to travel at a steady speed. The shopping trolley can be modelled as a particle.

(a) Show that the coefficient of friction is given by $(4 - 17 \sin \alpha) / (17 \cos \alpha)$.

(b) Given that $\tan \alpha = 13/84$, show that the coefficient of friction is 1/12.

(c) The same trolley, containing shopping, is now pushed up the slope. A pushing force of 100 N is required. Find the weight of the shopping in the trolley.

1.4A.11 A cart is on the point of slipping down a rough plane inclined at an angle of 30° to the horizontal. The coefficient of friction between the cart and the plane is 2/9. The cart is held in position by a light inextensible cord inclined at 15° to the plane, as shown in the diagram. The cart is modelled as a particle with a weight of 210 N. Find the magnitude of the tension in the cord.

1.4A.12 A rope inclined at an angle θ to the horizontal is used to pull a crate of bricks at a steady speed along a rough horizontal path. The tension in the rope is half the magnitude of the weight of the crate. The coefficient of friction between the crate and the path is μ.

(a) Show that μ is given by $(\cos \theta) / (2 - \sin \theta)$.

(b) Given that $\cos\theta = \sqrt{5} / 3$, find the value of μ.

1.4A.13 A particle of weight W N on a rough plane experiences a horizontal force of X N. The particle is on the point of slipping up the plane. The plane is inclined at an angle of α to the horizontal and the coefficient of friction is given by $\cos\alpha$. Given, further, that $3W = X$,

(a) show that $10\cos^4\alpha + \cos^2\alpha = 6\cos\alpha + 1$

(b) verify that $30° < \alpha < 31°$.

Exam-style questions · Chapter 1 review

R1.1 A wardrobe is at rest on a rough slope inclined at θ to the horizontal, where $\sin\theta = 5/13$. The wardrobe is on the point of slipping down the slope. Find the coefficient of friction between the wardrobe and the slope.

R1.2 A particle is acted upon by three horizontal forces, P N, Q N and 25 N, as shown in the diagram. The Q N force acts due east and the 25 N force acts due south. The P N force acts on a bearing of 325°. Given that the particle is in equilibrium, find:

(a) the value of P

(b) the value of Q.

R1.3 A peg bag of weight 1.8 N hangs in equilibrium from a washing line (made from thick rope). The rope on either side of the bag hangs at angles of 5° and 7° to the horizontal. Find the tension in each part of the rope.

R1.4 Angelene is pushing a small shopping trolley of weight 70 N through a supermarket at a steady speed. The floor of the supermarket is horizontal and the coefficient of friction between the trolley and the floor is 1/12. The force, P N, exerted by Angelene makes an angle of 8° to the horizontal.

(a) Find an expression for the reaction force in terms of P.

(b) Find an expression for the frictional force in terms of P.

(c) Find the value of P.

R1.5 Three forces, F, G and H, act upon a particle in a horizontal plane. Force F has a magnitude of 24 N and acts along the positive x-axis. Force G has a magnitude of 16 N and acts at an angle of 130° to the positive x-axis as shown in the diagram. The three forces are in equilibrium.

(a) Find the magnitude of H.

(b) Find the angle that H makes with the positive x-axis.

R1.6 A book of weight 6 N lies on a smooth plane inclined at β to the horizontal, where tan β = 3/4. The book is held in equilibrium by a horizontal force of magnitude G N. Calculate the value of G.

R1.7 Force J of magnitude 6 N acts along O_x (the positive x-axis) and force K of magnitude 9 N acts along O_y (the positive y-axis). Force L of magnitude 10 N acts at 50° to O_x, as shown in the diagram. Find:

(a) the magnitude of the resultant force

(b) the angle the resultant force makes with O_x.

R1.8 A particle of weight W N is held in position on a rough slope by a horizontal force of 40 N. The coefficient of friction between the particle and the slope is 1/3. The slope is inclined at an angle of 35° to the horizontal. The particle is on the point of slipping up the slope. Find the value of W.

R1.9 A block is on the point of slipping down a rough slope. The block weighs 50 N and is held in position by a horizontal force of 20 N. The slope is inclined at an angle of θ to the horizontal. The coefficient of friction between the block and the slope is 0.1.

(a) Show that $\tan \theta = 25/48$.

(b) Find the magnitude of the frictional force.

R1.10 A 24 N load is suspended from two identical cables, as shown in the diagram. The cables each make an angle of θ with the horizontal, where tan θ = 3/4. Find the tension in each of the cables.

R1.11 Force U of magnitude 5 N acts along O_x and force V of magnitude 7 N acts at an angle of 60° to O_x, as shown in the diagram. The resultant of U and V is R.

(a) Find the magnitude of R.

(b) When a third force X is added, acting along O_y, the new resultant force R_2 makes an angle of 50° with O_x. Find the magnitude of X.

(c) Find the magnitude of R_2.

R1.12 Four horizontal forces of magnitudes F N, F N, X N and 36 N act in equilibrium at a single point as shown in the diagram (F along Ox, F at 125° to Ox, X perpendicular to the second F, and 36 N at 60° below Ox).

(a) Find the value of F.

(b) Find the value of X.

R1.13 A toy helicopter is about to slip down a rough plank inclined at 20° to the horizontal. The coefficient of friction between the plank and the toy is 0.18. A horizontal force of 8 N is holding the helicopter in place.

(a) Find the weight of the toy helicopter.

(b) Given that the coefficient of friction is the same when the plank is moved, find the minimum horizontal force required to prevent the helicopter from slipping when the plank is inclined at 30°.

R1.14 A lawnmower of weight W N sits at rest on a grass slope inclined at an angle of θ to the horizontal. The lawnmower cord is also inclined at an angle of θ to the slope. The grass slope is rough and the coefficient of friction between the lawnmower and the grass is μ. The lawnmower is on the point of slipping down the grass slope.

(a) Show that the tension in the cord is given by $W(\mu \cos \theta - \sin \theta) / (\mu \sin \theta - \cos \theta)$.

(b) Given that the tension is 15 N, the weight is 80 N and θ is 20°, find the value of μ.

R1.15 A block of weight 50 N is resting on a rough slope inclined at an angle of α to the horizontal. There is a horizontal force X N acting upon the block such that the block is on the point of sliding down the slope. The coefficient of friction between the block and the slope is μ. Show that $X = (50(\tan \alpha - \mu)) / (1 + \mu \tan \alpha)$.

R1.16 A particle of weight 12 N is held at rest on a rough plane by a force, P, acting at 25° to the plane. The plane is inclined at 15° to the horizontal. The coefficient of friction between the particle and the plane is 0.32. The particle is about to slip up the plane. Find the magnitude of P.

R1.17 A particle of weight W N is held in limiting equilibrium on a plane inclined at θ to the horizontal, where sin θ = 3/5, by a force X N acting along the line of greatest slope of the plane. The coefficient of friction between the particle and the plane is μ. When the particle is on the point of slipping up the plane, X = 42 N. When the particle is on the point of slipping down the plane, X = 30 N.

(a) Find the value of W.

(b) Find the value of μ.

R1.18 A rock of weight W N is about to slip down a rough slope inclined at an angle of θ to the horizontal. The rock is held in position by a horizontal force of magnitude P N, where 5P = W. The coefficient of friction between the rock and the slope is 2/7. Show that tan θ = 17/33.

R1.19 A trolley of W N is held in limiting equilibrium on a rough plane inclined at an angle 30° to the horizontal. The trolley is on the point of rolling down the plane. The force preventing the trolley from slipping, P N, is also inclined at 30° to the horizontal, as shown in the diagram.

(a) Show that the coefficient of friction is given by $(\sqrt{3}\,(W - P)) / (3\,(W + P))$.

(b) Hence show that W > P.

R1.20 A bead of mass 2.2 kg has been threaded onto a string of length 63 cm. The ends of the string are attached to a vertical pole at points A and B, where A is 45 cm above B. The bead is held in position by a horizontal force F N as shown in the diagram, dividing the string in the ratio 4:3 such that the part attached to A is the longer part. The tensions in each part are given by T_A N and T_B N, where $2T_A = 3T_B$.

(a) Show that T_A is 55 N.

(b) Find the value of F.

R1.21 A cupboard is being pushed up a rough ramp inclined at an angle θ to the horizontal, where sin θ = 0.28. The pushing force, P N, is directed parallel to the line of greatest slope up the ramp. The coefficient of friction between the cupboard and the ramp is given by μ. When the cupboard is on the point of slipping down the plane, the pushing force is 5 N. When the cupboard is on the point of slipping up the plane, the pushing force is 65 N.

(a) Find the weight of the cupboard.

(b) Hence show that $\mu = 1/4$.

R1.22 A parcel of weight W N is pulled at a steady speed across rough horizontal ground by a rope of tension T N. The coefficient of friction between the parcel and the ground is μ and the rope is inclined at angle α to the horizontal. When $\sin\alpha = 12/13$, $\mu = 5/23$.

(a) Show that $35T = 13W$.

(b) Given that the tension is the same, find μ when $\sin\alpha = 5/13$.

R1.23 A table of weight W N is at rest on a rough plane. A force of P N, parallel to the line of greatest slope of the plane, is applied to the table such that the table is on the point of slipping up the slope. The value of P and the coefficient of friction, μ, between the table and the plane remain constant when the plane is angled at each of α and β to the horizontal where $\sin\alpha = 3/5$ and $\sin\beta = 8/17$.

(a) Show that $P = 13W / 7$.

(b) Hence show that $\mu = a/b$, where a and b are coprime and $a + b = 18$.

R1.24 A block of weight W N is pulled at a steady speed across a rough horizontal plane by a cord. The cord is inclined at an angle θ to the horizontal and has a tension of T N. The coefficient of friction between the block and the plane is given by μ.

(a) Show that $T = (\mu W) / (\cos \theta + \mu \sin \theta)$.

(b) When $\sin \theta = 3/5$, $T = k_1 W$. When $\sin \theta = 4/5$, $T = k_2 W$. Given that the ratio $k_1 : k_2 = 5/6$, find the value of μ.

R1.25 A theatre has a ramp for wheelchair access. The ramp is 30 cm tall and 11.5 m long.

(a) Find the angle of inclination of the ramp, correct to 1 decimal place.

(b) The total weight of a wheelchair and its user is 1.6 kN. It is pushed up the ramp at a steady speed. The coefficient of friction between the wheelchair and the ramp is 0.14. What is the minimum force that needs to be applied to the wheelchair to push it up the ramp?

(c) For practical reasons, it is decided that this force is too high and should be reduced to 200 N. It is suggested that a different material with a lower coefficient of friction is used for the surface of the ramp. Find the coefficient of friction between the wheelchair and surface of the ramp, given that the minimum force is to be 200 N.

Check yourself

CY.1 A sled of weight 150 N is pulled along horizontal snow at a steady speed by a rope inclined at 25° above the horizontal with tension 40 N. Calculate the normal reaction force R and the coefficient of friction μ.

CY.2 A packing crate of mass 45 kg sits on a ramp inclined at 20° to the horizontal. If the coefficient of static friction is $\mu = 0.42$, determine whether the crate will remain in equilibrium under gravity alone or slip down the ramp. (Take $g = 10\ \text{m s}^{-2}$.)

CY.3 Three coplanar forces of magnitudes 10 N, 14 N, and F N act at a point in equilibrium. The angle between the 10 N and 14 N forces is 115°. Use the cosine rule on the vector triangle to find F.

CY.4 A particle of weight 50 N is held on a rough plane inclined at 35° to the horizontal by a horizontal pushing force P. The coefficient of friction is μ = 0.25. Calculate the minimum value of P required to prevent the particle from slipping down the plane.

CY.5 A small pendant of weight 4 N hangs in equilibrium from two light strings making angles of 40° and 50° with a horizontal ceiling. Use Lami's theorem or resolving to find the tension in each string.

Chapter quiz

Q1 A contact force between two rough surfaces is resolved into two perpendicular components. What are they called?

A	B	C	D
Normal reaction and friction	Weight and tension	Thrust and friction	Centripetal force and reaction

Q2 A force F acts at angle θ above the horizontal. What is its resolved component in the horizontal direction?

A	B	C	D
F sin θ	F cos θ	F tan θ	F / cos θ

Q3 A particle is in equilibrium under coplanar forces. What must be true about the resolved components in any direction?

A	B	C	D
Their sum must equal the weight	The sum of squares must be zero	Their algebraic sum must equal zero	The horizontal sum must exceed vertical sum

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Q4 A block of weight W rests on a plane inclined at angle θ to horizontal. What is the weight component down the slope?

A	B	C	D
W cos θ	W tan θ	W / sin θ	W sin θ

Q5 A block of mass m rests on a smooth plane inclined at angle θ. What is the normal reaction R from the plane? (g = 10)

A	B	C	D
10m cos θ	10m sin θ	10m tan θ	10m / cos θ

Q6 What is the relationship between friction F, normal reaction R, and friction coefficient μ for any rough surface?

A	B	C	D
F = μR always	F ≤ μR	F ≥ μR	F = R / μ

Q7 A particle on a rough slope inclined at θ to the horizontal is on the point of slipping down. What is μ?

A	B	C	D
sin θ	cos θ	tan θ	1 / tan θ

Q8 What is the key physical distinction between tension in a light string and thrust in a light rod?

A	B	C	D
Tension pulls away from particle; thrust pushes towards it	Tension only acts vertically; thrust acts horizontally	Tension requires friction; thrust is frictionless	Tension is a vector; thrust is a scalar

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Q9 Two perpendicular forces of 5 N and 12 N act at a point. What is the magnitude of their resultant force?

A	B	C	D
17 N	7 N	60 N	13 N

Q10 Three coplanar forces hold a particle in equilibrium. When drawn tip-to-tail, what geometric shape do they form?

A	B	C	D
An open triangle	A closed triangle	A right angle only	A straight line only

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Q11 Under Lami's theorem for three forces P, Q, R in equilibrium, how are the magnitudes related to opposite angles?

A	B	C	D
$P / \sin\alpha = Q / \sin\beta = R / \sin\gamma$	$P \sin\alpha = Q \sin\beta = R \sin\gamma$	$P / \cos\alpha = Q / \cos\beta = R / \cos\gamma$	$P^2 = Q^2 + R^2 - 2QR\cos\alpha$

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/as-a-level-mathematics/m1-1-forces-and-equilibrium/